CS580:
MC Ray Tracing:
Part II, Importance Sampling

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(윤성의)

Course URL:
http://sglab.kaist.ac.kr/~sungeui/GCG

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Class Objectives:

- Importance sampling for:
  - Direct terms
  - Lights
  - Indirect terms

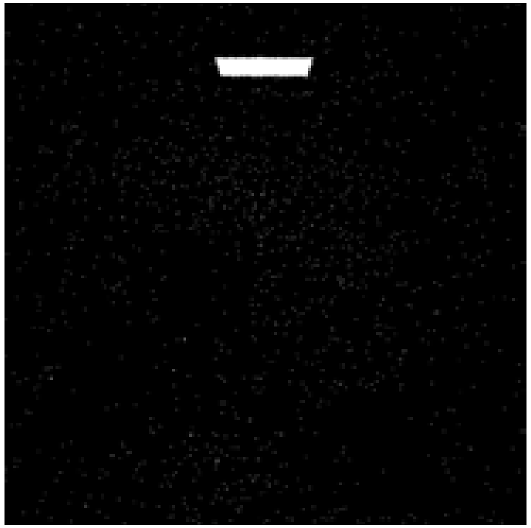
Performance and Error

- **Want better quality with smaller number of samples**
  - **Fewer samples → better performance**
  - **Stratified sampling**
  - **Quasi Monte Carlo: well-distributed samples**

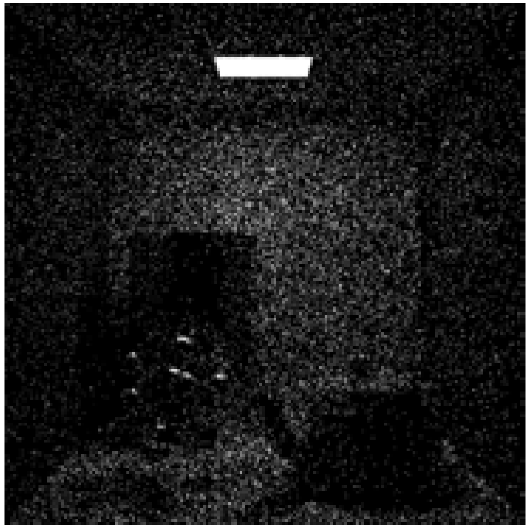
- **Faster convergence**
  - **Importance sampling: next-event estimation**

Path Tracing

Sample hemisphere



1 sample/pixel



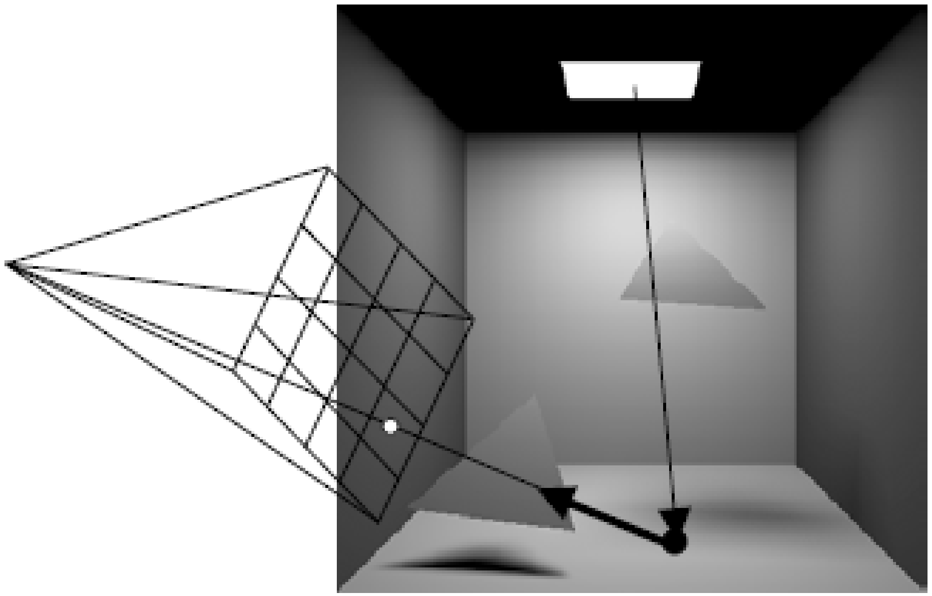
16 samples/pixel

256 samples/pixel

- Importance Sampling: compute direct illumination separately!

Direct Illumination

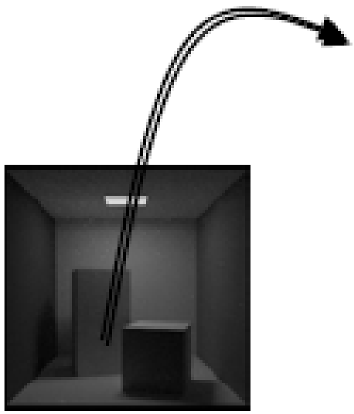
- Paths of length 1 only, between receiver and light source



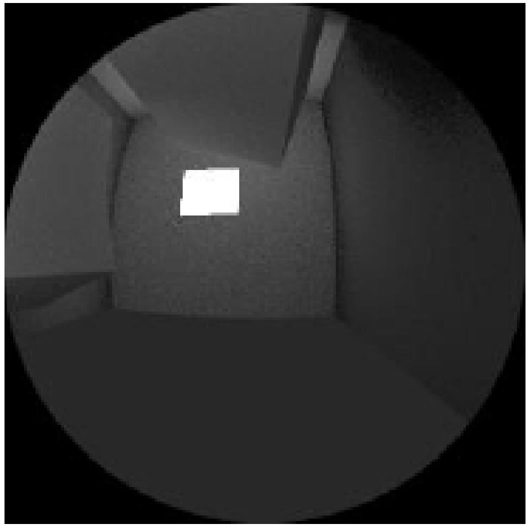
Importance Sampling

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos(\Psi, n_x) \cdot d\omega_\Psi$$

Radiance from light sources + radiance from other surfaces



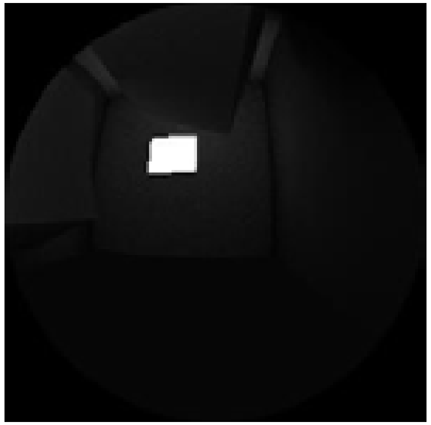
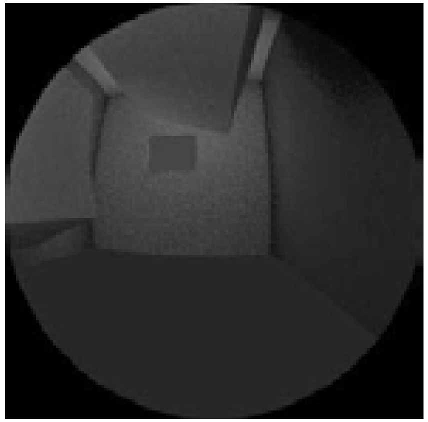
$$= L_e + \int_{\Omega_x}$$



$$\cdot f_r \cdot \cos$$

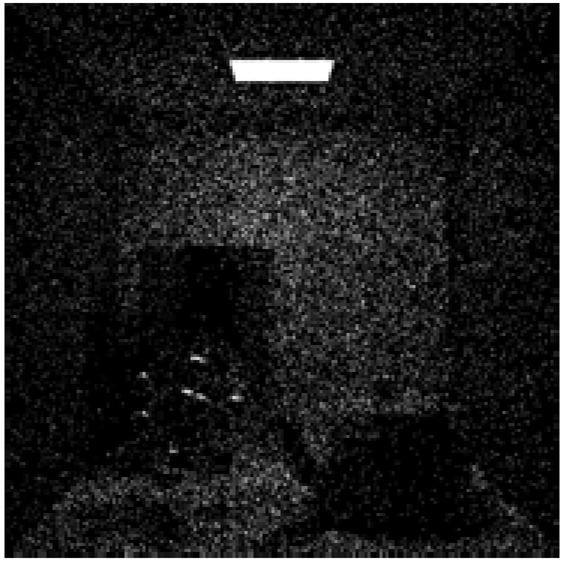
Importance Sampling

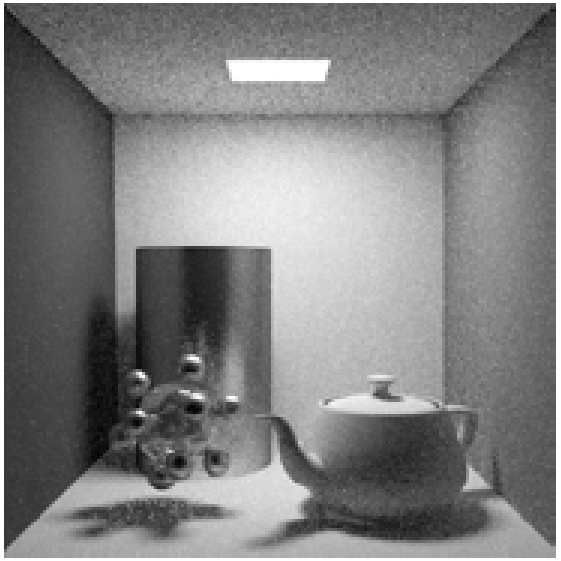
$$L(x \rightarrow \Theta) = L_e + L_{direct} + L_{indirect}$$

$$= L_e + \int_{\Omega_x}$$  $$\cdot f_r \cdot \cos + \int_{\Omega_x}$$  $$\cdot f_r \cdot \cos$$

- So … sample direct and indirect with separate MC integration

Comparison





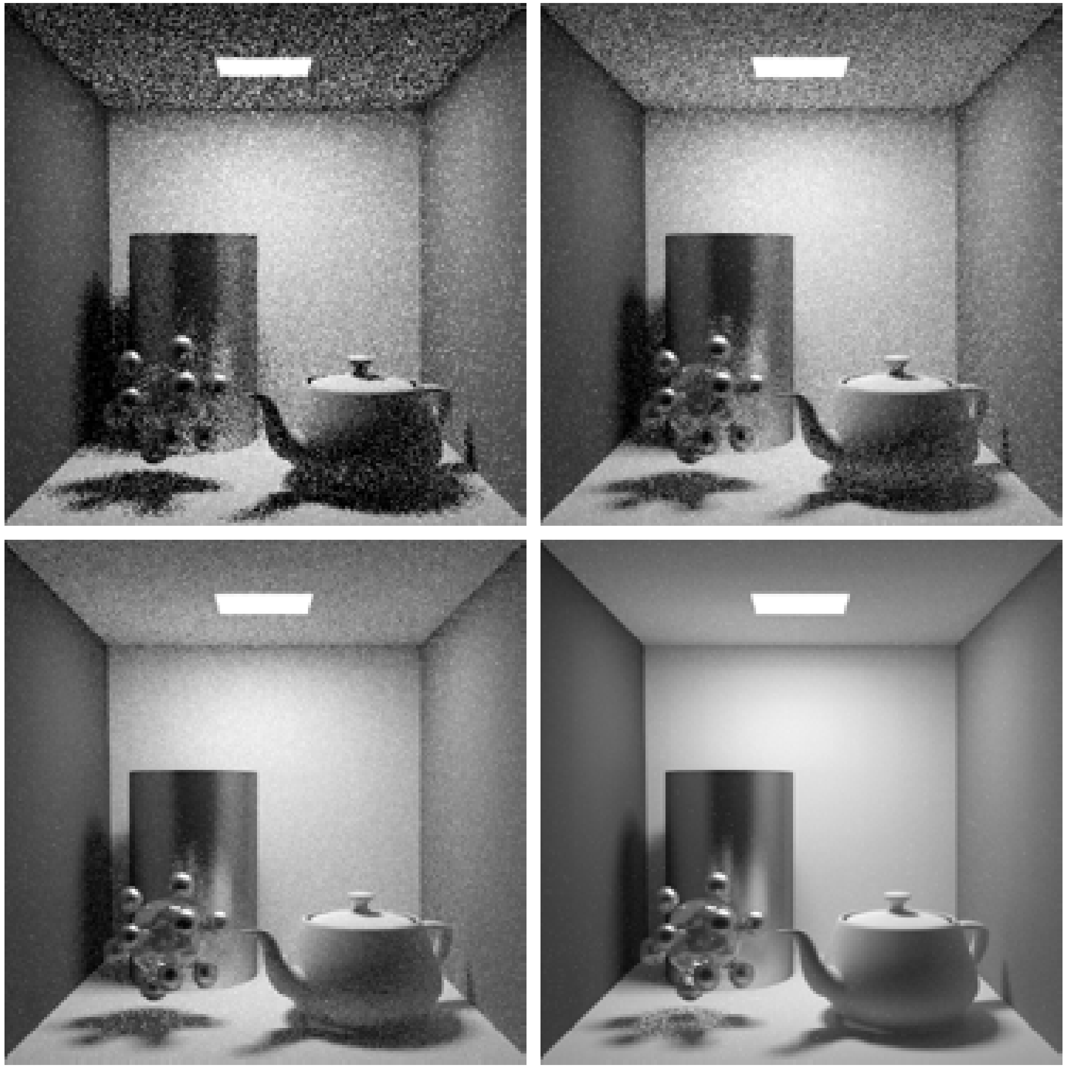
From kavita's slides

- **With and without considering direct illumination**
  - **16 samples / pixel**

Rays per pixel

1 sample/ pixel

4 samples/ pixel



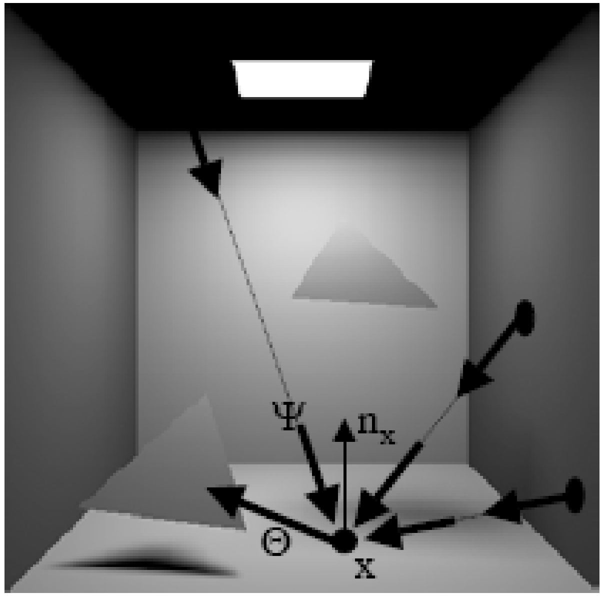
16 samples/ pixel

256 samples/ pixel

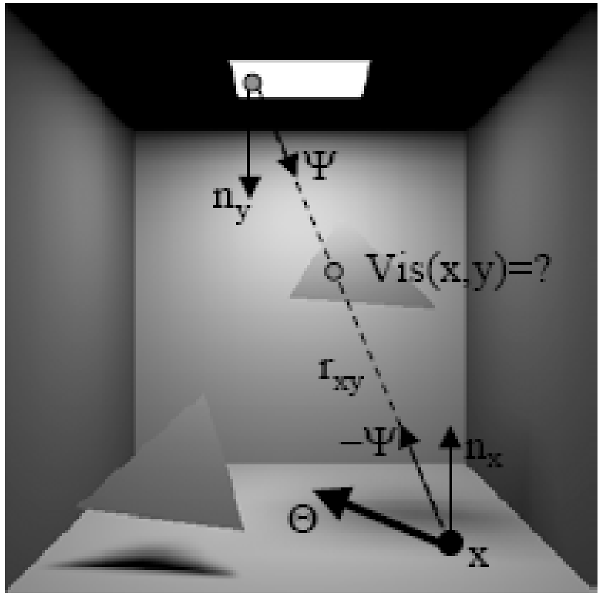
Direct Illumination

$$L(x \to \Theta) = \int_{A_{source}} f_r(x, -\Psi \leftrightarrow \Theta) \cdot L(y \to \Psi) \cdot G(x,y) \cdot dA_y$$

$$G(x,y) = \frac{\cos(n_x, \Theta)\cos(n_y, \Psi) Vis(x,y)}{r_{xy}^2}$$



hemisphere integration



area integration

Estimator for direct lighting

- Pick a point on the light's surface with pdf

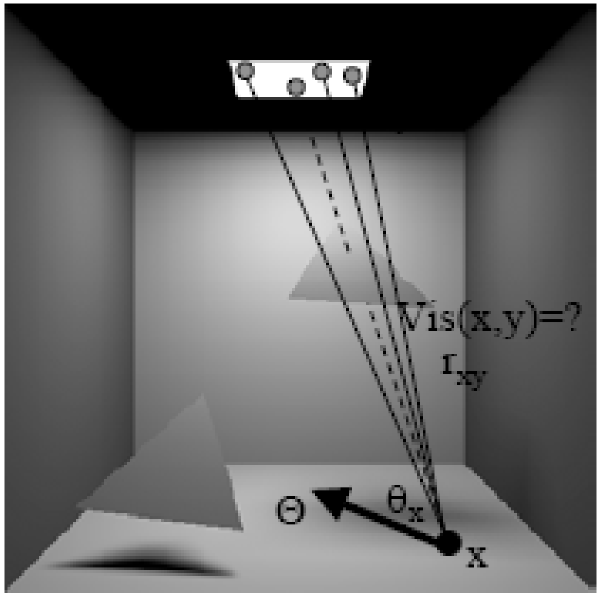
$$p(y)$$

- For N samples, direct light at point x is:

$$E(x) = \frac{1}{N} \sum_{i=1}^{N} \frac{f_r L_{source} \frac{\cos\theta_x \cos\theta_{\overline{y}_i}}{r_{x\overline{y}_i}^2} Vis(x, \overline{y}_i)}{p(\overline{y}_i)}$$

Generating direct paths

- Pick surface points y_i on light source
- Evaluate direct illumination integral



$$\langle L(x \to \Theta) \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{f_r(\ldots)L(\ldots)G(x, y_i)}{p(y_i)}$$

PDF for sampling light

- Uniform

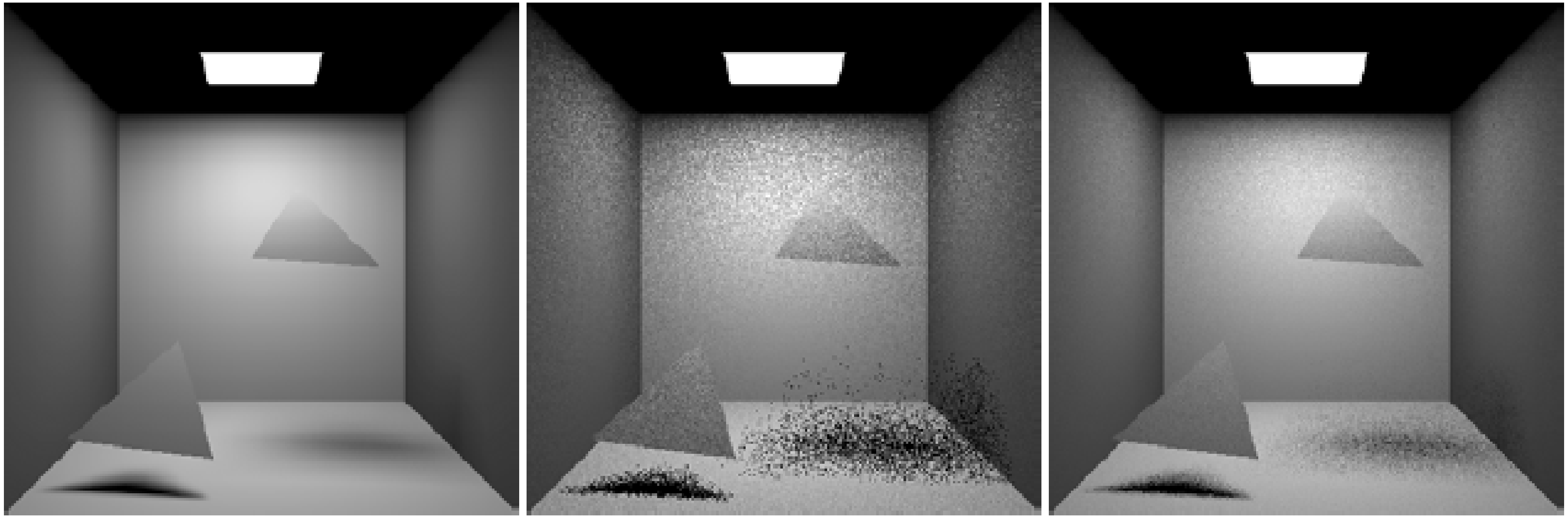
$$p(y) = \frac{1}{Area_{source}}$$

- Pick a point uniformly over light's area
  - Can stratify samples

- Estimator:

$$E(x) = \frac{Area_{source}}{N} \sum_{i=1}^{N} f_r L_{source} \frac{\cos\theta_x \cos\theta_{\bar{y}_i}}{r_{x\bar{y}_i}^2} Vis(x, \bar{y}_i)$$

More points ...

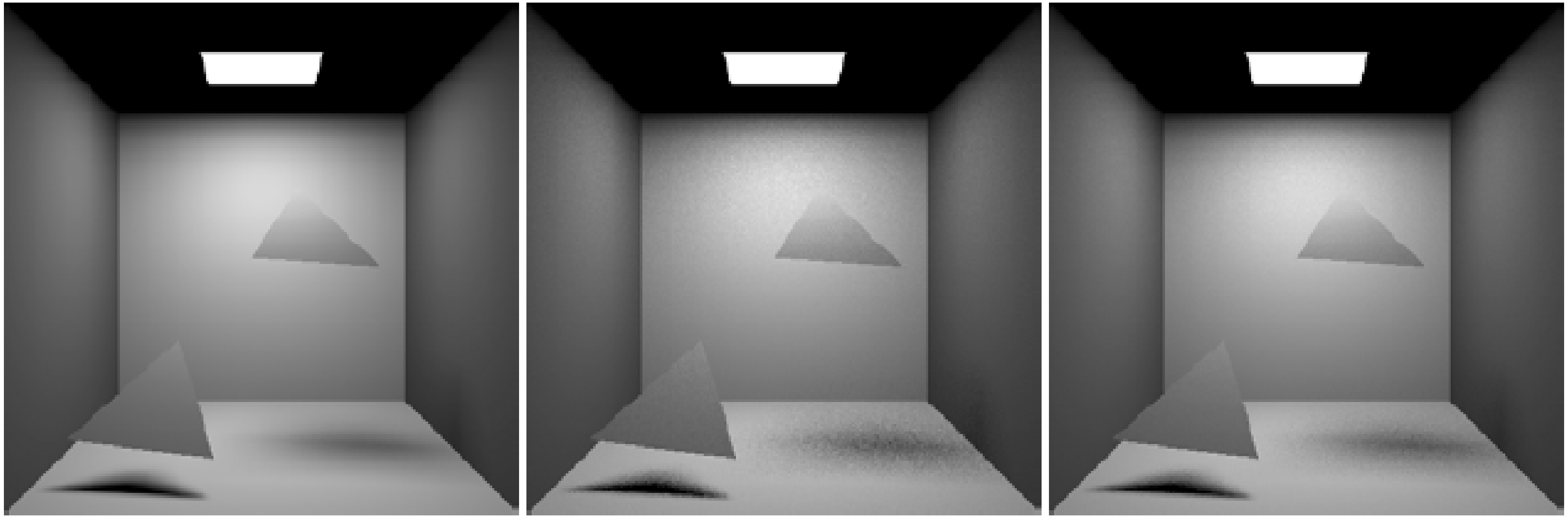


1 shadow ray

9 shadow rays

$$E(x) = \frac{Area_{source}}{N} \sum_{i=1}^{N} f_r L_{source} \frac{\cos\theta_x \cos\theta_{\bar{y}_i}}{r_{x\bar{y}_i}^2} Vis(x, \bar{y}_i)$$

Even more points ...



36 shadow rays

100 shadow rays

$$E(x) = \frac{Area_{source}}{N} \sum_{i=1}^{N} f_r L_{source} \frac{\cos\theta_x \cos\theta_{\bar{y}_i}}{r^2_{x\bar{y}_i}} Vis(x, \bar{y}_i)$$

Different pdfs

- Uniform

$$p(y) = \frac{1}{Area_{source}}$$

- Solid angle sampling
  - Removes cosine and distance from integrand
  - Better when significant foreshortening

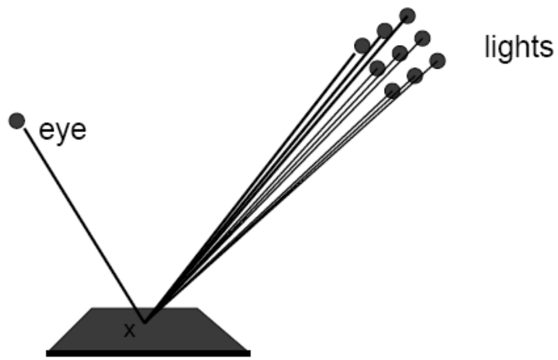
$$E(x) = \frac{1}{N} \sum_{i=1}^{N} \frac{f_r L_{source} \frac{\cos\theta_x \cos\theta_{\bar{y}_i}}{r_{x\bar{y}_i}^2} Vis(x, \bar{y}_i)}{p(\bar{y}_i)}$$

Parameters

- How to distribute paths within light source?
  - Uniform
  - Solid angle
  - What about light distribution?
- How many paths ("shadow-rays")?
  - Total?
  - Per light source? (~intensity, importance, …)

Scenes with many lights

- Many lights in scenes: M lights

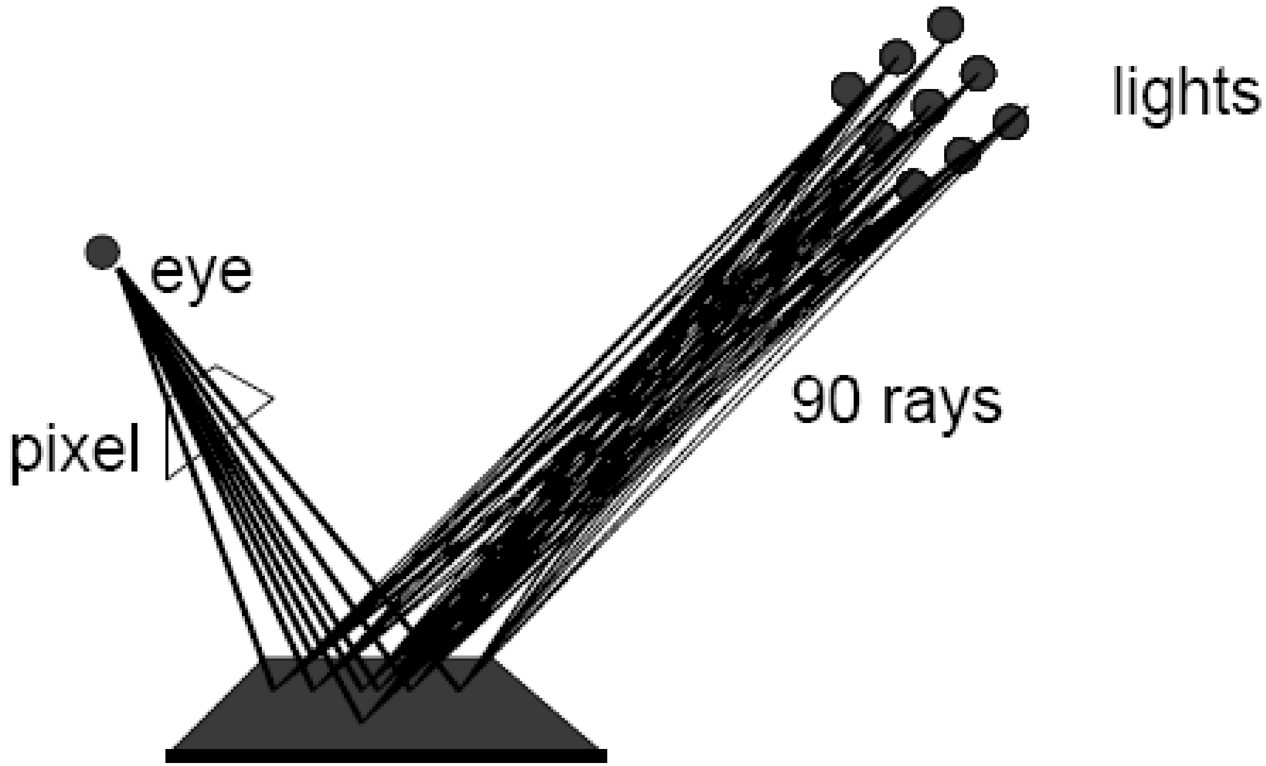


- How to handle many lights?
- Formulation 1: M integrals, one per light
  - Same solution technique as earlier for each light

$$L(x \to \Theta) = \sum_{i=1}^{M} \int_{A_{source}} f_r(x, -\Psi \leftrightarrow \Theta) \cdot L_{source}(y \to -\Psi) \cdot G(x, y) \cdot dA_y$$

Antialiasing: pixel

- Anti-aliasing



Formulation over all lights

- When M is large, each direct lighting sample is very expensive
- We would like to importance sample the lights
- Instead of M integrals

$$L(x \to \Theta) = \sum_{i=1}^{M} \int_{A_{source}} f_r(x, -\Psi \leftrightarrow \Theta) \cdot L_{source}(y \to -\Psi) \cdot G(x, y) \cdot dA_y$$

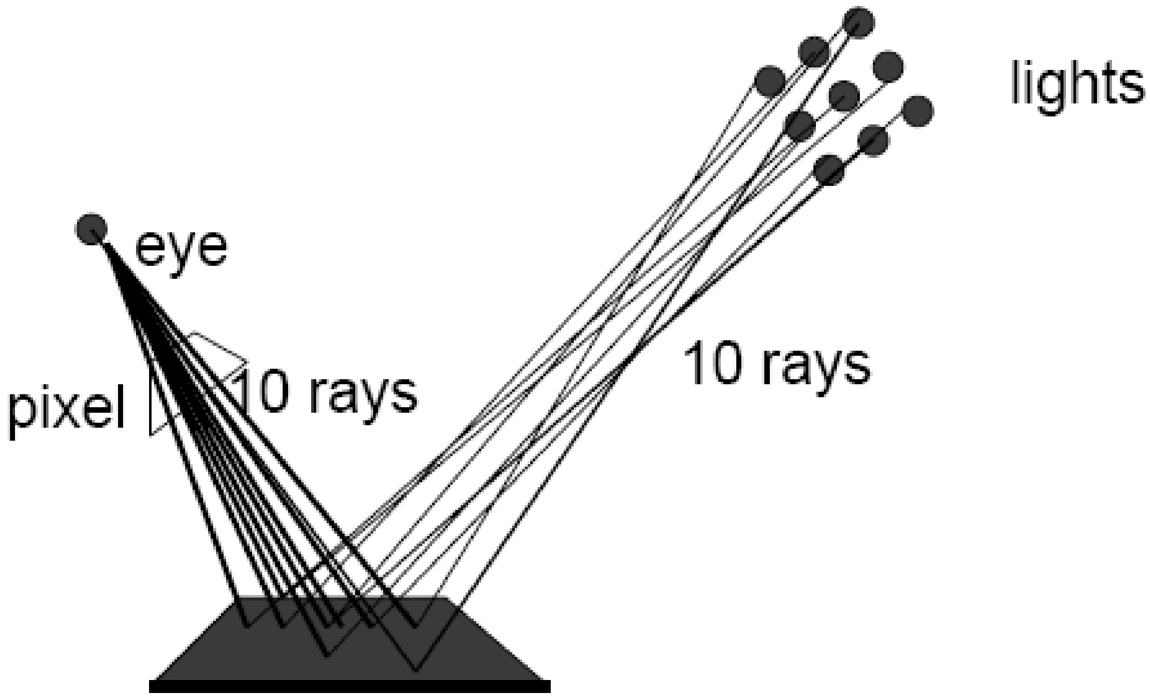
- Formulation over 1 integration domain

$$L(x \to \Theta) = \int_{A_{all\ lights}} f_r(x, -\Psi \leftrightarrow \Theta) \cdot L_{source}(y \to -\Psi) \cdot G(x, y) \cdot dA_y$$

Why?

- Do not need a minimum of M rays/sample
- Can use only one ray/sample
- Still need N samples, but 1 ray/sample
- Ray is distributed over the whole integration domain
  - Can importance sample the lights

Anti-aliasing



From kavita's slides

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How to sample the lights?

- A discrete pdf $p_L(k_i)$ picks the light k_i
- A surface point is then picked with pdf $p(y_i|k_i)$
- Estimator with N samples:

$$E(x) = \frac{1}{N}\sum_{i=1}^{N}\frac{f_r L_{source} G(x, \bar{y}_i)}{p_L(k_i)\, p(y_i \mid k_i)}$$

Strategies for picking light

– Uniform $p_L(k) = \frac{1}{M}$

– Area $p_L(k) = \frac{A_k}{\sum A_k}$

– Power $p_L(k) = \frac{P_k}{\sum P_k}$

Do not take visibility into account!

Research on Many Lights

- **Ward 91**
  - **Sort lights based on their maximum contribution**
  - **Pick bright lights based on a threshold**
  - **Do not consider visibility**
- **Many other papers**
- **One of recent works:**
  - **LightCuts: A Scalable Approach to Illumination, SIG. 05, Walter et al.**

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Direct paths

- Different path generators produce different estimators and different error characteristics
- Direct illumination general algorithm:

```
compute_radiance (point, direction)
        est_rad = 0;
        for (i=0; i<n; i++)
                p = generate_path;
                est_rad += energy_transfer(p) / probability(p);
        est_rad = est_rad / n;
        return(est_rad);
```

Stochastic Ray Tracing

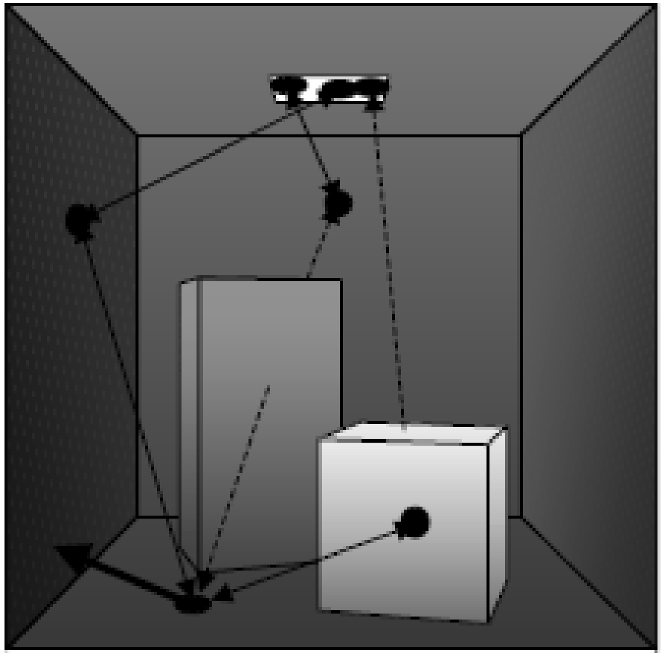
- Sample area of light source for direct term
- Sample hemisphere with random rays for indirect term
- Optimizations:
  - Stratified sampling
  - Importance sampling
  - Combine multiple probability density functions into a single PDF

Indirect Illumination

- Paths of length > 1
- Many different path generators possible
- Efficiency depends on:
  - BRDFs along the path
  - Visibility function
  - ...

Indirect paths - surface sampling

- Simple generator (path length = 2):
  - select point on light source
  - select random point on surfaces



- per path:
  - 2 visibility checks

Indirect paths - surface sampling

- Indirect illumination (path length 2):

y → z → x

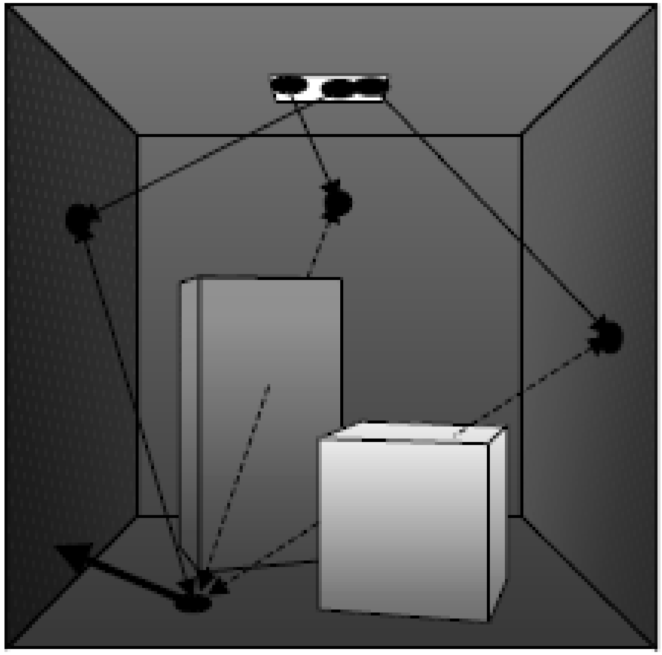
$$L(x \to \Theta) = \int_{A_{source}} \int_{A} L(y \to \Psi_1) f_r(z, -\Psi_1 \leftrightarrow \Psi_2) G(z, y) f_r(x, -\Psi_2 \leftrightarrow \Theta) G(z, x) dA_z dA_y$$

$$\langle L(x \to \Theta) \rangle = \frac{1}{N} \sum_{i=1}^{N} \frac{L(y_i \to \Psi_{1i}) f_r(z_i, -\Psi_{1i} \leftrightarrow \Psi_{2i}) G(z_i, y_i) f_r(x, -\Psi_{2i} \leftrightarrow \Theta) G(z_i, x)}{p_y(y_i) p_z(z_i)}$$

- 2 visibility values cause noise
  - which might be 0

Indirect paths - source shooting

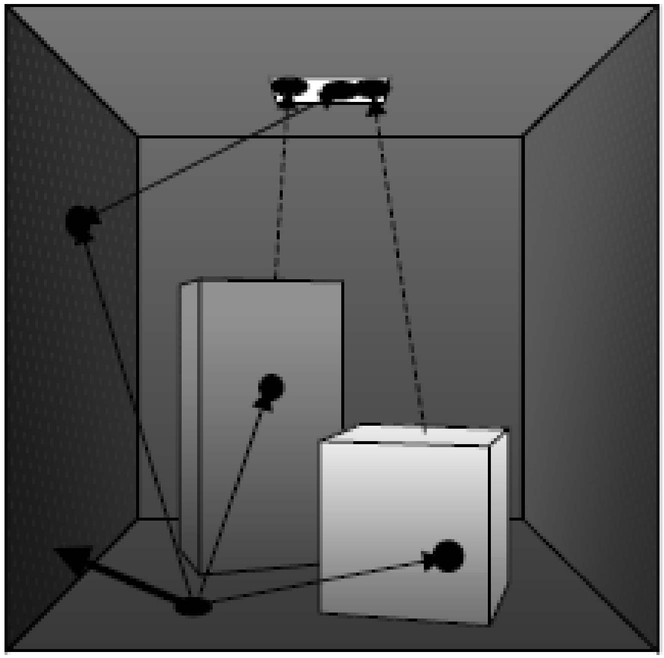
- Shoot ray from light source, find hit location
- Connect hit point to receiver



– per path:
  - 1 ray intersection
  - 1 visibility check

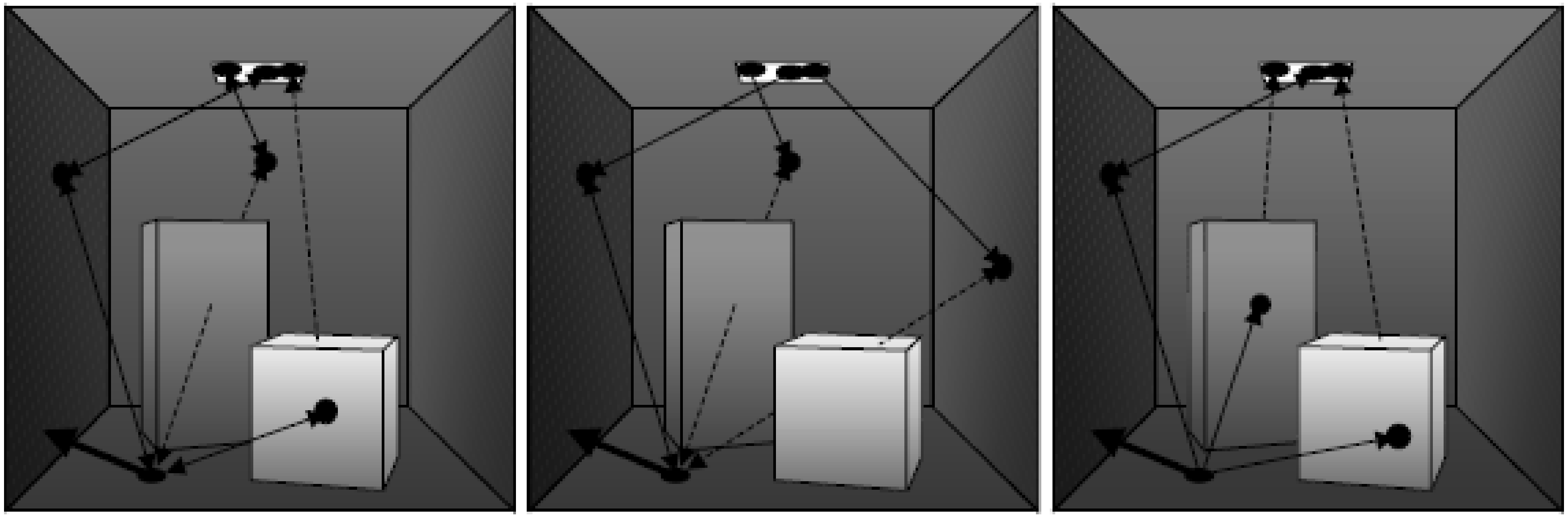
Indirect paths - receiver gathering

- Shoot ray from receiver point, find hit location
- Connect hit point to random point on light source



– per path:
  - 1 ray intersection
  - 1 visibility check

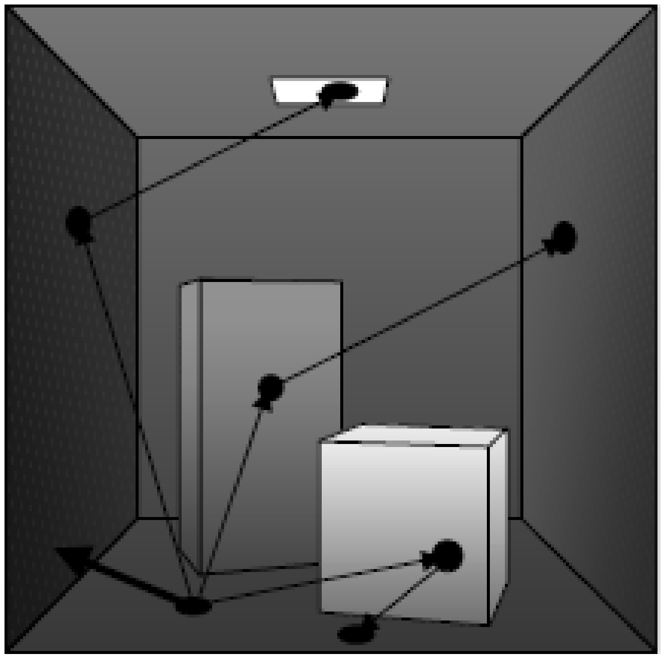
Indirect paths



Surface sampling	Source shooting	Receiver gathering
- 2 visibility terms; can be 0	- 1 visibility term - 1 ray intersection	- 1 visibility term - 1 ray intersection

More variants …

- Shoot ray from receiver point, find hit location
- Shoot ray from hit point, check if on light source



– per path:
  - 2 ray intersections
  - L_e might be zero

Indirect paths

- Same principles apply to paths of length > 2
  - generate multiple surface points
  - generate multiple bounces from light sources and connect to receiver
  - generate multiple bounces from receiver and connect to light sources
  - …
- Estimator and noise characteristics change with path generator

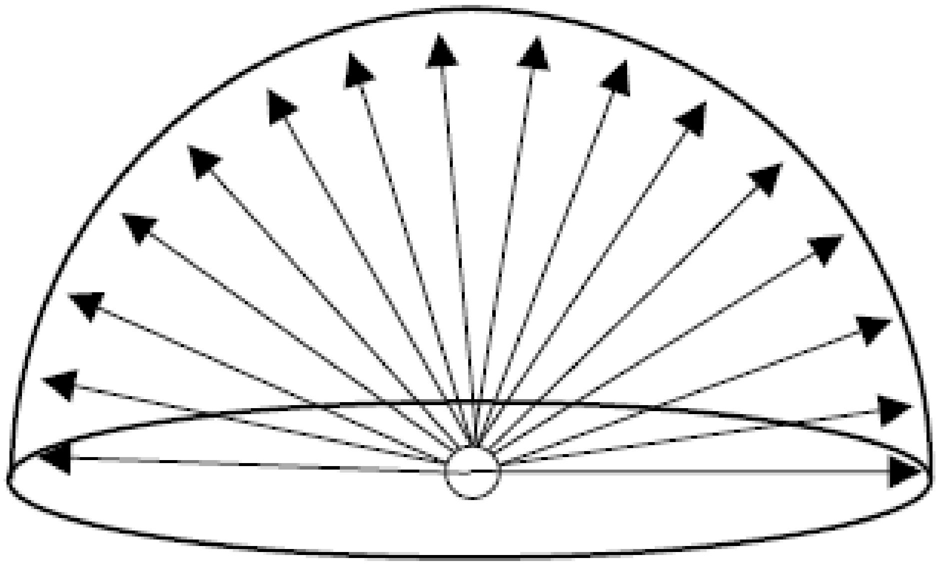
Stochastic Ray Tracing

- Sample area of light source for direct term
- Sample hemisphere with random rays for indirect term
- Optimizations:
  - Stratified sampling
  - Importance sampling
  - Combine multiple probability density functions into a single PDF

Sampling strategies

- Uniform sampling over the hemisphere

$$L(x \to \Theta) = \int_{\Omega_x} L(x \leftarrow \Psi) \cdot f_r(\Psi \leftrightarrow \Theta) \cdot \cos(\Psi, n_x) \, d\omega_\Psi$$

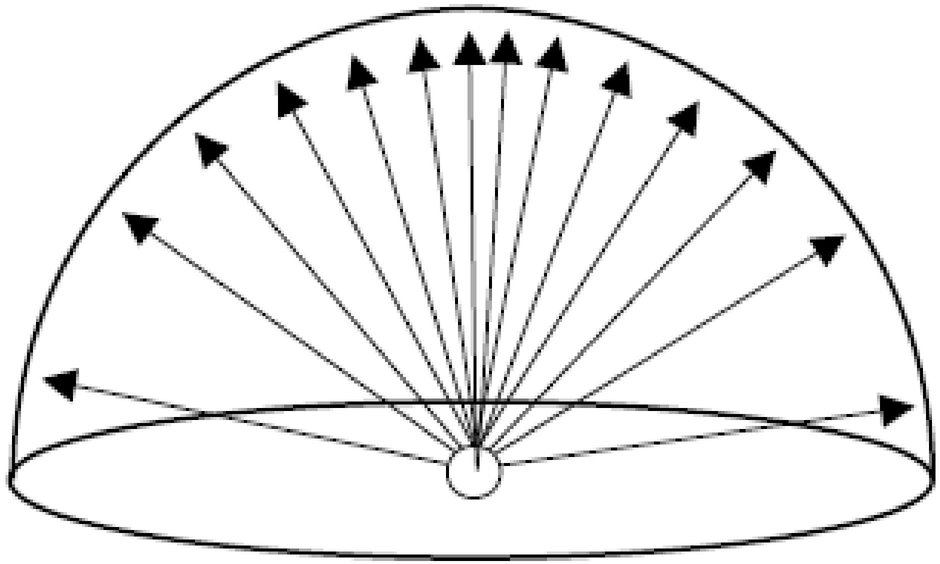


$$p(\Theta) = 1/(2\pi)$$

Sampling strategies

- Sampling according to the cosine factor

$$L(x \to \Theta) = \int_{\Omega_x} \underline{L(x \leftarrow \Psi) \cdot f_r(\Psi \leftrightarrow \Theta)} \cdot \cos(\Psi, n_x) \cdot d\omega_\Psi$$

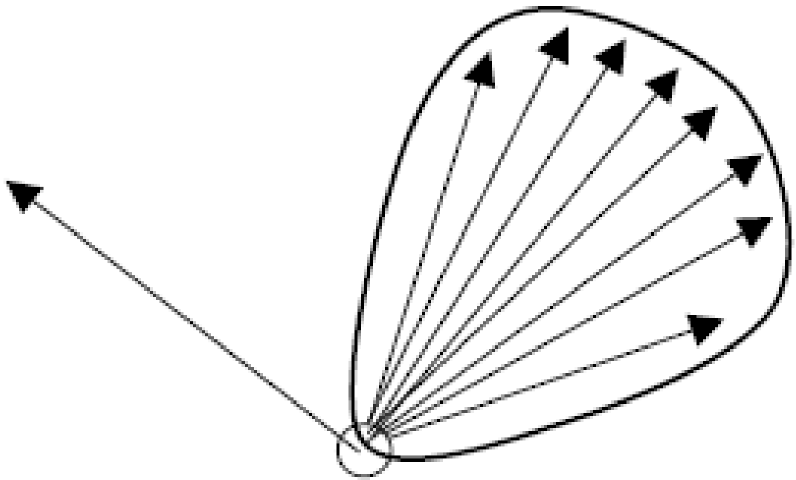


$$p(\Theta) = \cos\theta / \pi$$

Sampling strategies

- Sampling according to the BRDF

$$L(x \to \Theta) = \int_{\Omega_x} \underline{L(x \leftarrow \Psi)} \cdot f_r(\Psi \leftrightarrow \Theta) \cdot \underline{\cos(\Psi, n_x)} \cdot d\omega_\Psi$$

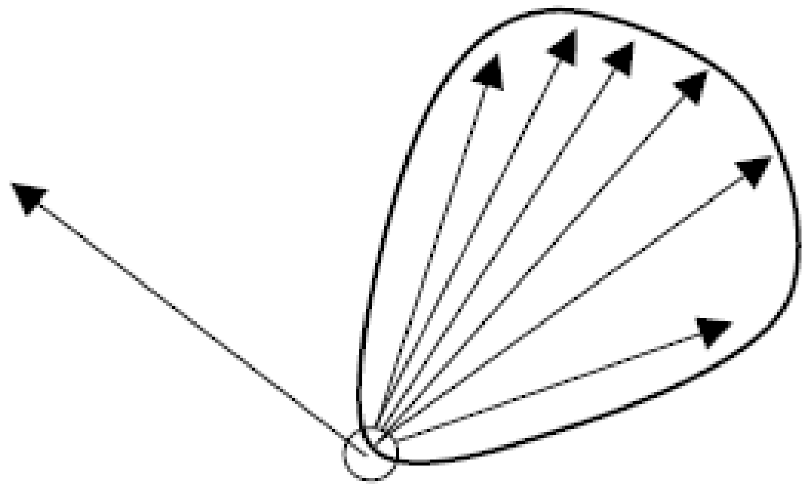


$$p(\Theta) \sim f_r(\Theta \leftrightarrow \Psi)$$

Sampling strategies

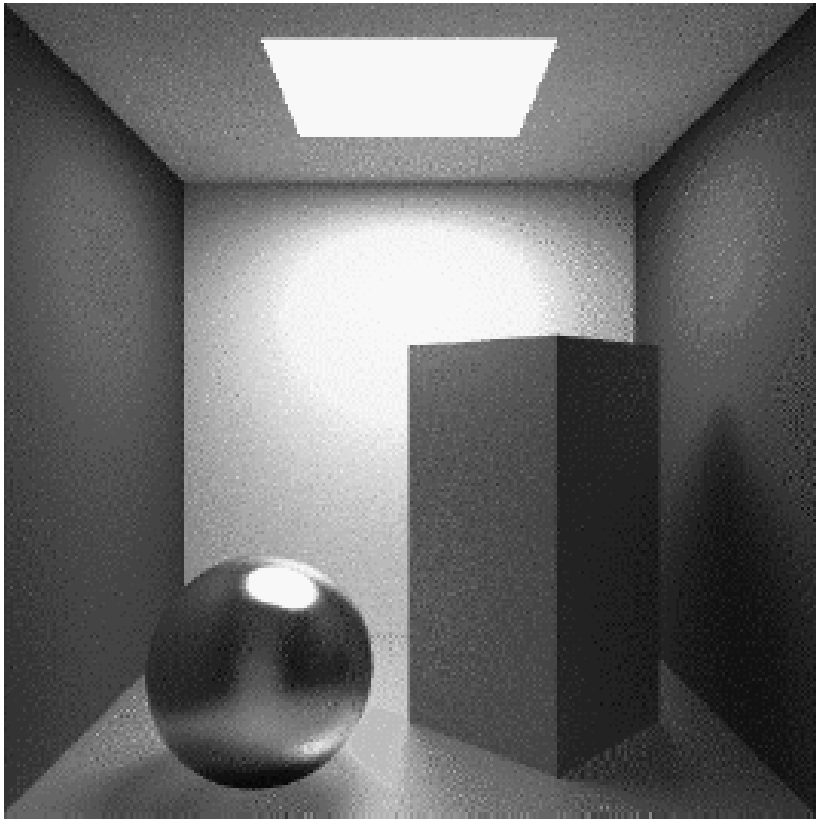
- Sampling according to the BRDF times the cosine

$$L(x \to \Theta) = \int_{\Omega_x} \underline{L(x \leftarrow \Psi)} \cdot f_r(\Psi \leftrightarrow \Theta) \cdot \cos(\Psi, n_x) \cdot d\omega_\Psi$$

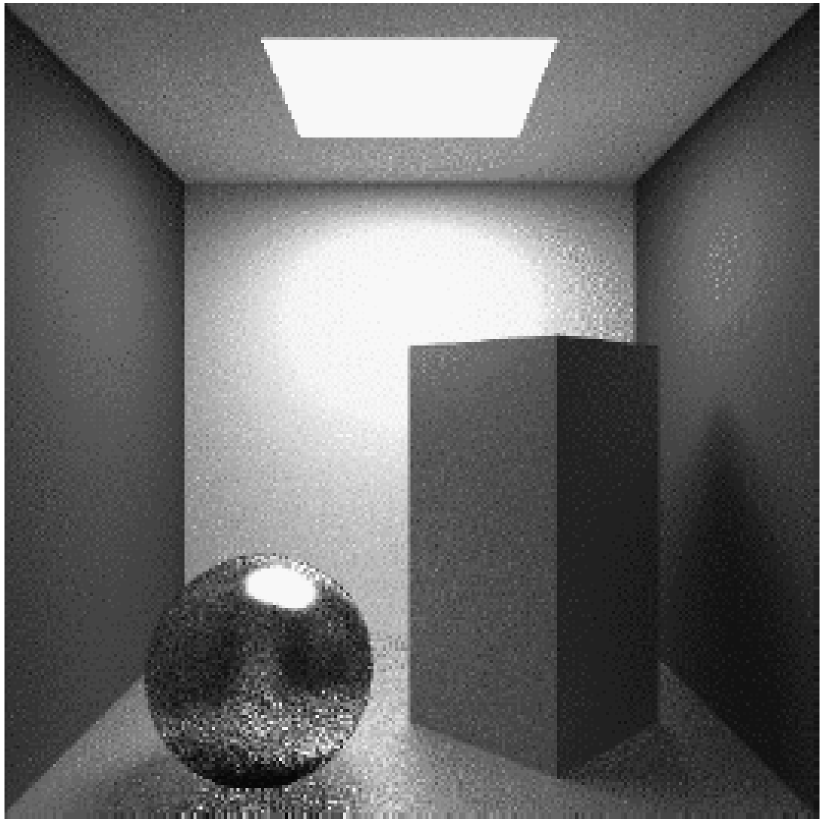


$$p(\Theta) \sim f_r(\Theta \leftrightarrow \Psi) \cos\theta$$

Comparison



With importance sampling
(brdf on sphere)



Without importance sampling

General GI Algorithm

- Design path generators
- Path generators determine efficiency of GI algorithm
- Black boxes
  - Evaluate BRDF, ray intersection, visibility evaluations, etc

Class Objectives were:

- Importance sampling for:
  - Direct terms
  - Lights
  - Indirect terms