1. Anisotropic Gaussian Mutations for Metropolis Light Transport through Hessian-Hamiltonian Dynamics

2. Fusing State Spaces for Markov Chain Monte Carlo Rendering

20193168 Kim Hyunsu

Overview

1. **Backgrounds**
   - Metropolis Algorithm
   - Metropolis Light Transport
   - Primary Sample Space MLT
2. **Anisotropic Gaussian Mutations for Metropolis Light Transport through Hessian-Hamiltonian Dynamics**
   - Introduction
   - Results
3. **Fusing State Spaces for Markov Chain Monte Carlo Rendering**
   - Introduction
   - Results

Metropolis Algorithm [N. Metropolis ‘53, W. K. Hastings ‘70]

- One kind of **Markov chain Monte Carlo** methods.
- Generate samples of target probability distribution by sampling **arbitrary distribution**.

Metropolis Algorithm [N. Metropolis ‘53, W. K. Hastings ‘70]

Target $r(x)/Z$. Symmetric proposal $q(x'|x)$.

1. initialise x_1 randomly; $n:=1$
2. repeat:
   a) $x' \sim q(x'|x_n)$; $\alpha := \min(1, r(x')/r(x_n))$
   b) $u \sim \text{uniform}(0,1)$
   c) $x_{n+1} := (\text{if } (u \leq \alpha) \text{ then } x' \text{ else } x_n)$; $n:=n+1$

Metropolis Light Transport [E. Veach ‘97]

- **Metropolis Algorithm** to compute **light transport integral**.
- **Mutation strategies** to avoid **variance** originated from sampling from proposal distribution.

Metropolis Light Transport [E. Veach ‘97]

The light transport equation

$$L(\mathbf{x}' \to \mathbf{x}'') = L_{\mathrm{e}}(\mathbf{x}' \to \mathbf{x}'') + \int_{\mathcal{M}} L(\mathbf{x} \to \mathbf{x}')\, f_{\mathrm{s}}(\mathbf{x} \to \mathbf{x}' \to \mathbf{x}'')\, G(\mathbf{x} \leftrightarrow \mathbf{x}')\, dA(\mathbf{x}). \tag{6}$$

$$G(\mathbf{x} \leftrightarrow \mathbf{x}') = V(\mathbf{x} \leftrightarrow \mathbf{x}')\, \frac{|\cos(\theta_{\mathrm{o}})\ \cos(\theta'_{\mathrm{i}})|}{\|\mathbf{x} - \mathbf{x}'\|^2}$$

Metropolis Light Transport [E. Veach '97]

The measurement equation

$$m_j = \int_{\mathcal{M}^2} L_e(\mathbf{x}_0 \to \mathbf{x}_1)\, G(\mathbf{x}_0 \leftrightarrow \mathbf{x}_1)\, W_e^{(j)}(\mathbf{x}_0 \to \mathbf{x}_1)\, dA(\mathbf{x}_0)\, dA(\mathbf{x}_1)$$
$$+ \int_{\mathcal{M}^3} L_e(\mathbf{x}_0 \to \mathbf{x}_1)\, G(\mathbf{x}_0 \leftrightarrow \mathbf{x}_1)\, f_s(\mathbf{x}_0 \leftrightarrow \mathbf{x}_1 \leftrightarrow \mathbf{x}_2)$$
$$G(\mathbf{x}_1 \leftrightarrow \mathbf{x}_2)\, W_e^{(j)}(\mathbf{x}_1 \to \mathbf{x}_2)\, dA(\mathbf{x}_0)\, dA(\mathbf{x}_1)\, dA(\mathbf{x}_2)$$
$$+ \cdots . \qquad (8)$$

$$m_j = \int_{\Omega} f_j(\bar{x})\, d\mu(\bar{x})$$

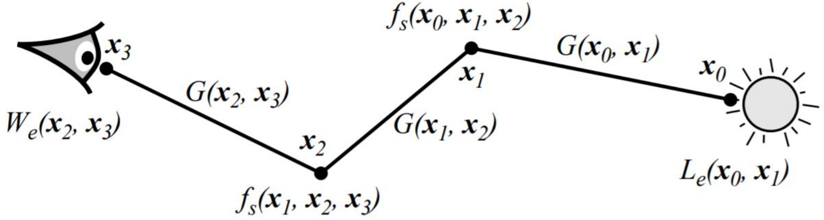


Figure 1: The measurement contribution function f_j is a product of many factors (shown for a path of length 3).

Metropolis Light Transport [E. Veach '97]

Goal: Estimate the expectation of the measurement with Monte Carlo

$$m_j = E\left[\frac{1}{N}\sum_{i=1}^{N}\frac{w_j(\bar{X}_i)f(\bar{X}_i)}{p(\bar{X}_i)}\right]$$

where $p = (1/b)\, f$

Metropolis algorithm in Metropolis Light Transport

- Sampling $p = (1/b)\, f$ using **Metropolis algorithm**.
- **Flexible** to choose the proposal probability distribution (mutation).
- **Mutation** strategies: bidirectional mutations, perturbations, and lens subpath mutations.
- Mutations are constructed in the **path space**.

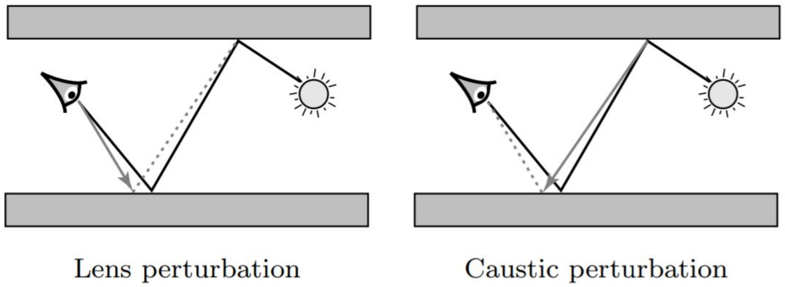


Figure 3: The lens edge can be perturbed by regenerating it from either side: we call these *lens perturbations* and *caustic perturbations*.

Metropolis Light Transport - Pseudo code

$$
\begin{aligned}
&\bar{x} \leftarrow \textsc{InitialPath}() \\
&image \leftarrow \{\text{ array of zeros }\} \\
&\textbf{for } i \leftarrow 1 \textbf{ to } \mathrm{N} \\
&\qquad \bar{y} \leftarrow \textsc{Mutate}(\bar{x}) \\
&\qquad a \leftarrow \textsc{AcceptProb}(\bar{y}|\bar{x}) \\
&\qquad \textbf{if } \textsc{Random}() < a \\
&\qquad\quad \textbf{then } \ \bar{x} \leftarrow \bar{y} \\
&\qquad \textsc{RecordSample}(image, \bar{x}) \\
&\textbf{return } image
\end{aligned}
$$

Pros and Cons

- **Pros** It is efficient for images that the **bidirectional method fails**.
  - We can **treat complicated functions** according to the mutation strategies.
- **Cons** Too **complex** to implement and **not flexible** enough.
  - Consider **proper mutation strategy** depended on environments.

Primary Sample Space MLT [C. Kelemen ‘02]

- Original MLT mutates samples in **path space**.
- Primary Sample Space MLT mutates samples in **primary sample space**.

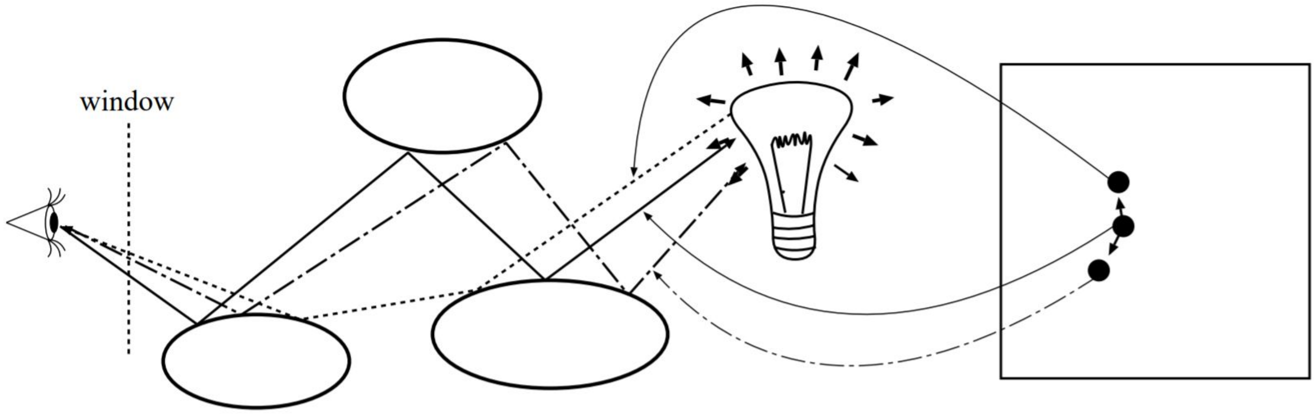
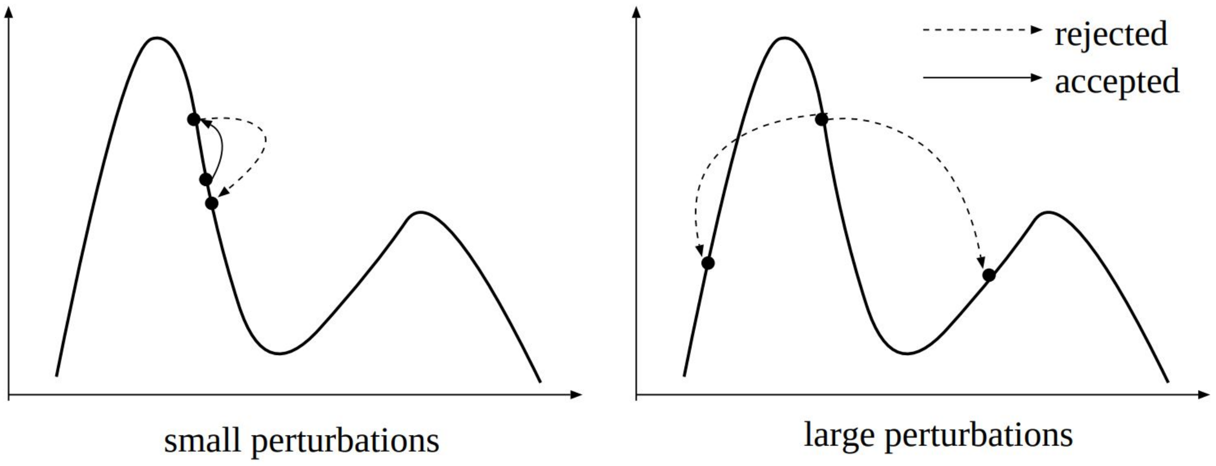


Figure 3: *The correspondance between the mutations in the primary sample space and in the path space*

Primary Sample Space MLT [C. Kelemen '02]

- In original MLT, the new sample **largely perturbed around a peak** of the function is easy to be **rejected**.
- because the **rejection probability** is proportional to the **ratio of the importances**.



Primary Sample Space MLT [C. Kelemen '02]

- So, we need to make the function relatively **flat**.
- By **transformation** to a simple space such as **high-dimensional uniform distribution** → **primary sample space**.
- We can just do **importance sampling** in the primary sample space.

$\mathbf{u} = T(\mathbf{z})$: Transformation from path space to primary sample space

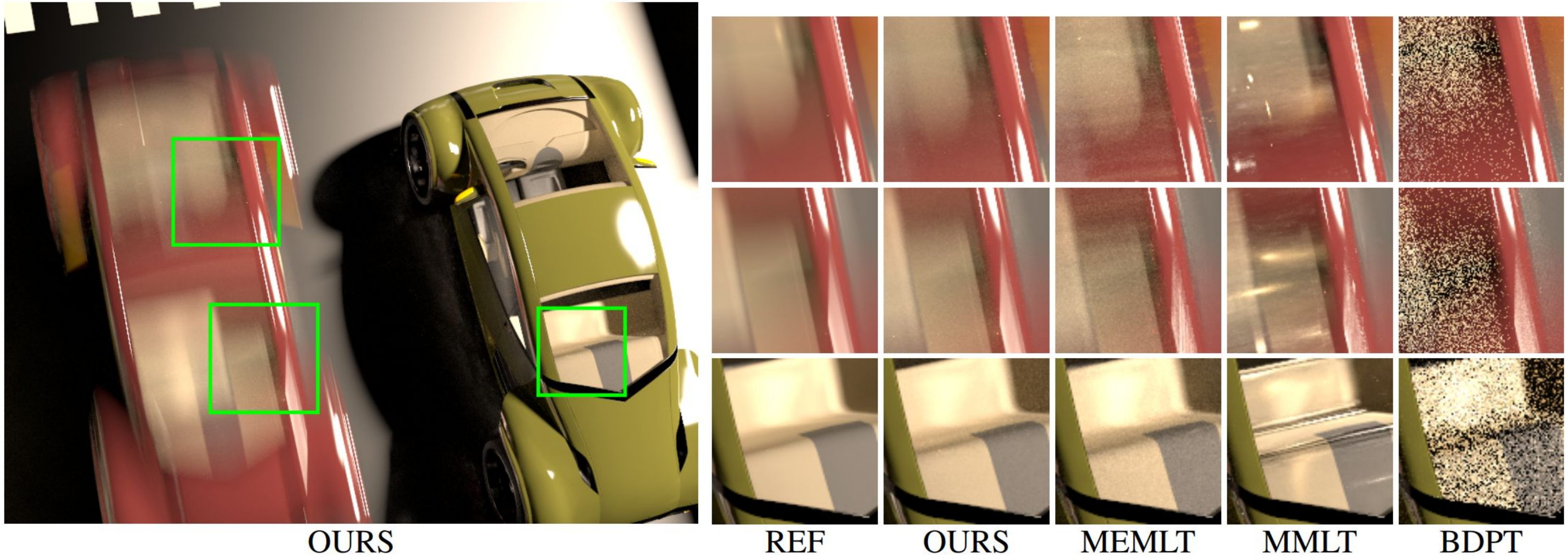
$$L^*(\mathbf{u}) = L(T^{-1}(\mathbf{u})) \cdot \left| \frac{dT^{-1}(\mathbf{u})}{d\mathbf{u}} \right| = \frac{L(T^{-1}(\mathbf{u}))}{t(\mathbf{u})}$$ where $$\left| \frac{dT^{-1}(\mathbf{u})}{d\mathbf{u}} \right| = \frac{1}{t(\mathbf{u})}$$

Importance function $$\boxed{I^*(\mathbf{u}) = \frac{I(T^{-1}(\mathbf{u}))}{t(\mathbf{u})}.}$$

Pros and Cons

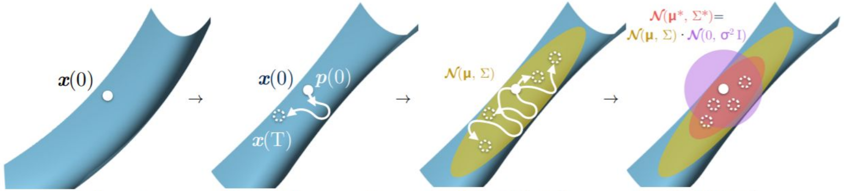
- **Pros** Achieve **low variance** by **high acceptance ratio**.
    - They make the importance function flat to reduce ratio of the importances.
- **Cons** Still difficult in complex function such as **highly glossy area**.
    - Even more difficult than the original MLT.

Anisotropic Gaussian Mutations for Metropolis Light Transport through Hessian-Hamiltonian Dynamics [T.-M. Li ‘15]



Anisotropic Gaussian Mutations for Metropolis Light Transport through Hessian-Hamiltonian Dynamics [T.-M. Li '15]

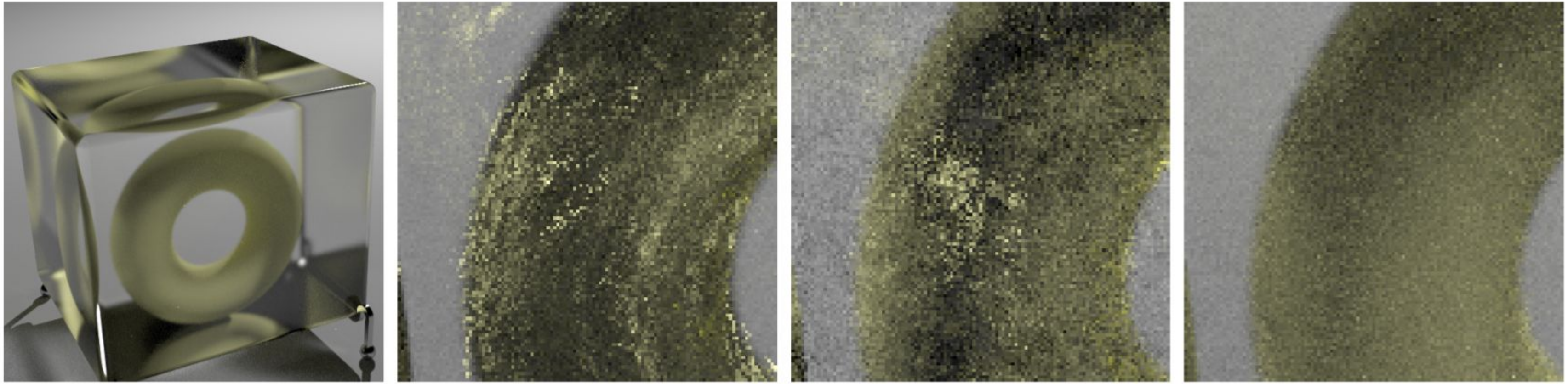
- Inspired from **Hamiltonian Monte Carlo**, one kind of Markov Chain Monte Carlo methods.
- Captures the strong **anisotropy** of the light transport integrand using **Hessian** of measurement contribution function.



(a) *original target function and sample position* **(b)** *simulate Hamiltonian dynamics on local quadratic approximation* **(c)** *proposals result in a Gaussian distribution* **(d)** *multiply with a prior to limit variance and draw proposals from red*

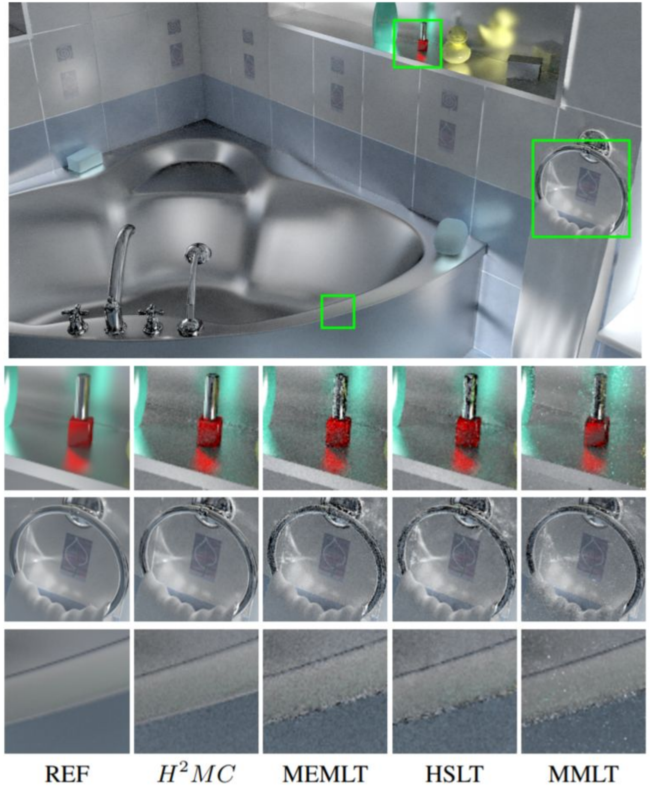
- Efficient for rendering **difficult light transport** such as highly-glossy transport (multi-bounce) combined with motion blur.

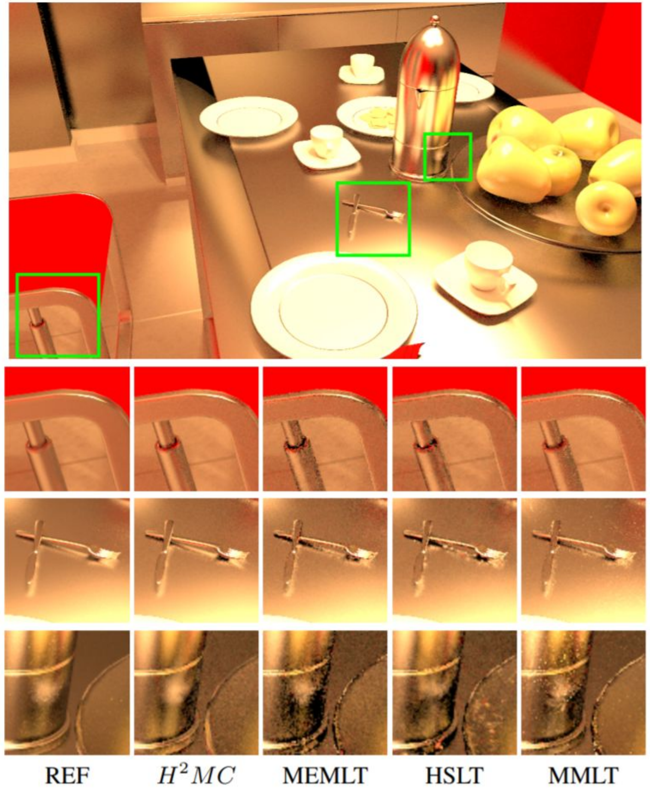
Anisotropic Gaussian Mutations for Metropolis Light Transport through Hessian-Hamiltonian Dynamics [T.-M. Li '15]



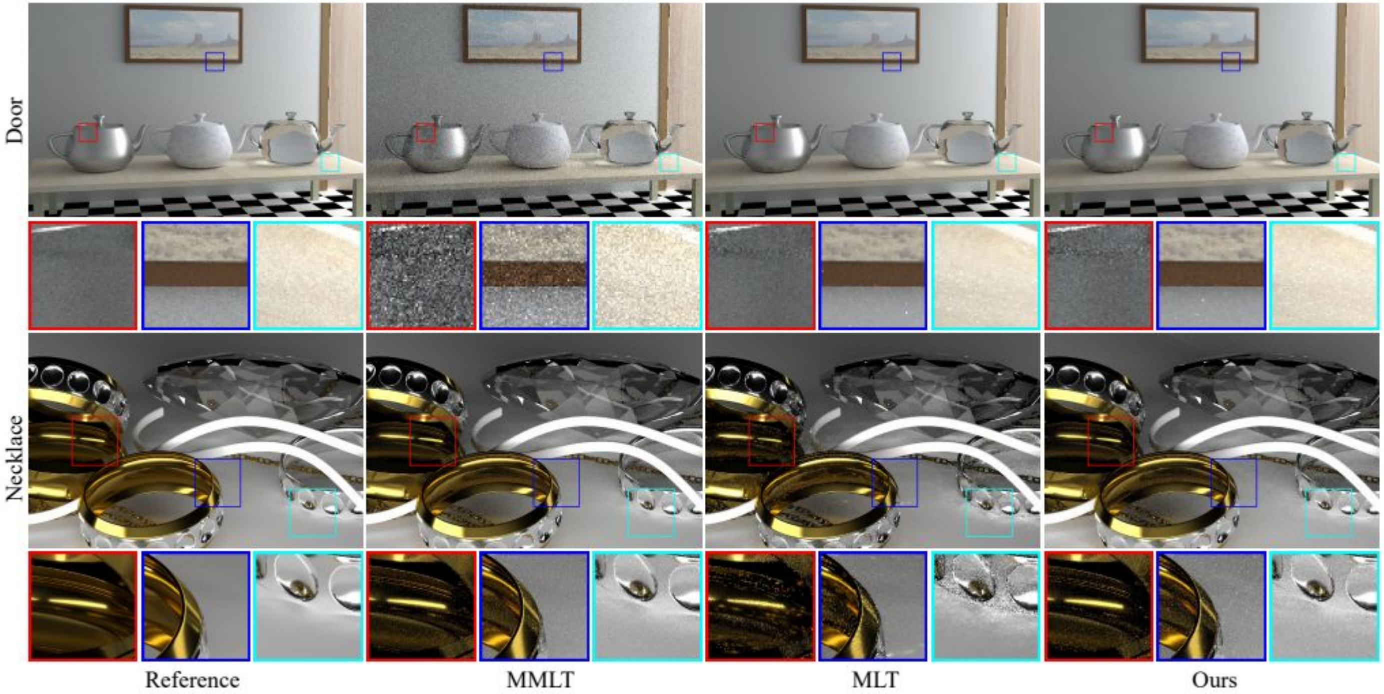
(a) TORUS **(b)** *Kelemen et al., isotropic proposal* **(c)** *Ours, isotropic proposal* **(d)** *Ours, H^2MC proposal*

Equal time comparison



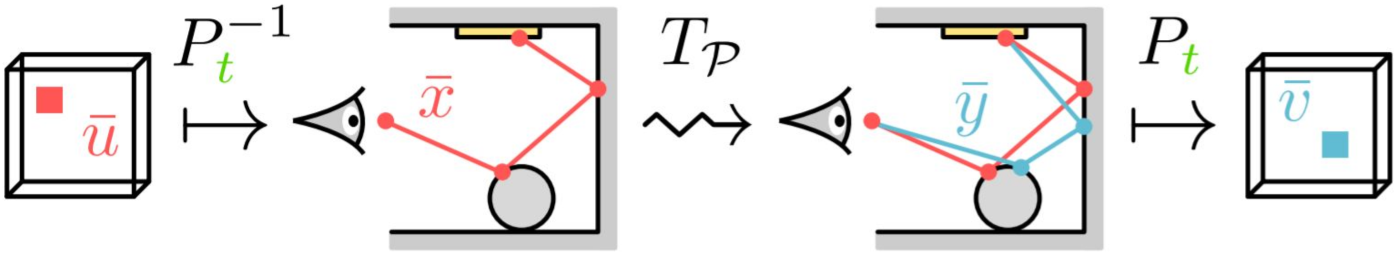


Fusing State Spaces for Markov Chain Monte Carlo Rendering [H. Otsu '17]

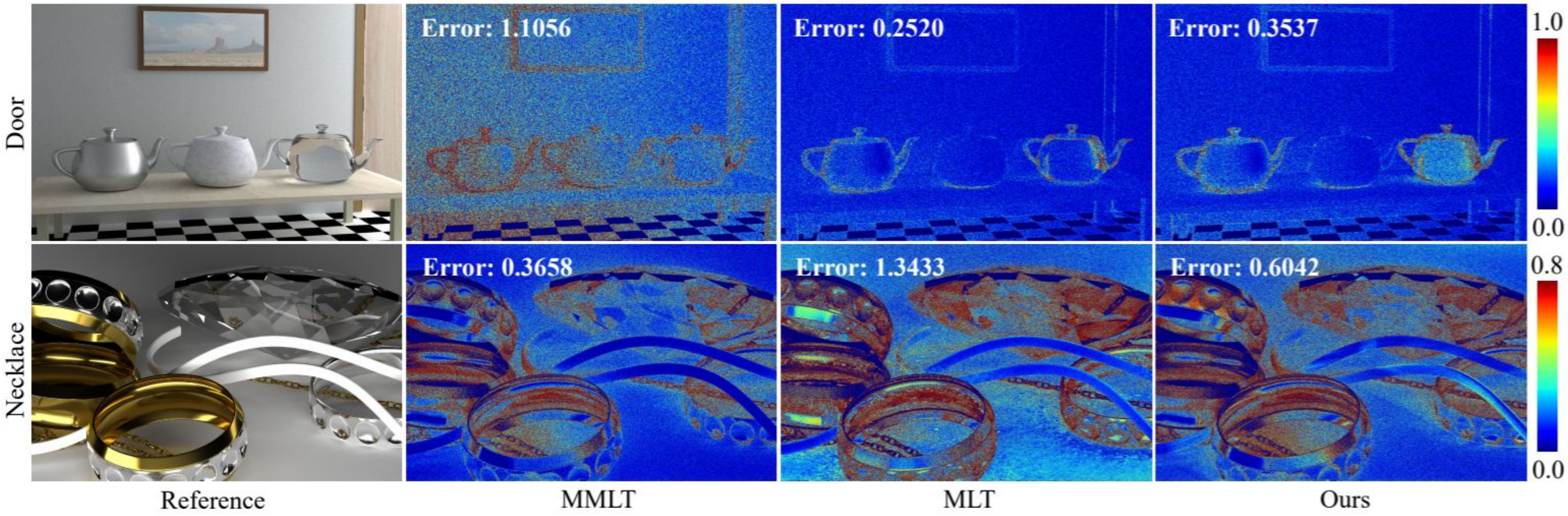


Fusing State Spaces for Markov Chain Monte Carlo Rendering [H. Otsu '17]

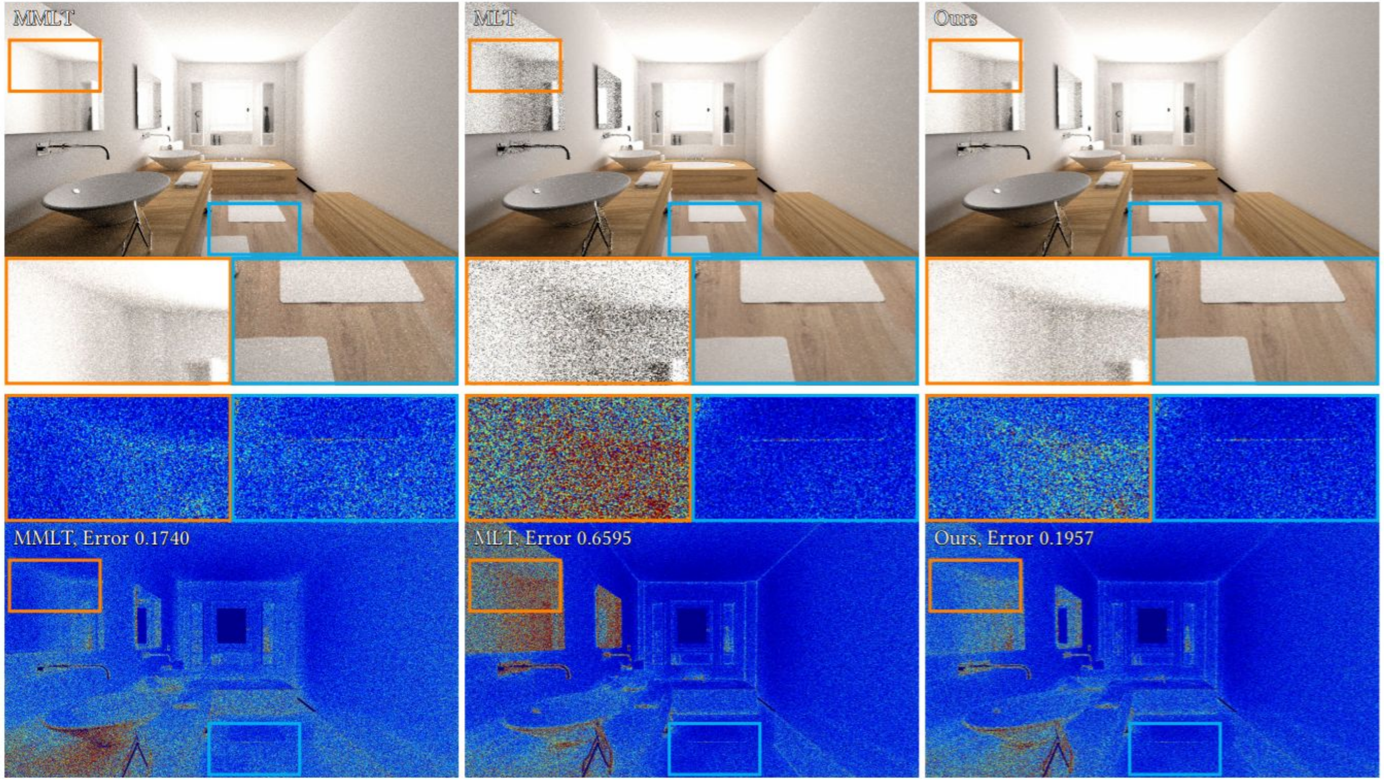
- **Fused** mutations in the **path space** and the **primary sample space**.
- Possible to use **mutation strategies** designed for **one space** with another **mutation strategies** designed for respective **other space**.
- In the paper, they combined **Manifold Exploration MLT** (path space) and **Multiplexed MLT** (primary sample space)



Pixel-wise relative error



Pixel-wise relative error



Conclusions

1. Anisotropic Gaussian Mutations for Metropolis Light Transport through Hessian-Hamiltonian Dynamics
    a. Captures anisotropic rendering using Hessian
    b. Good for difficult light transports

2. Fusing State Spaces for Markov Chain Monte Carlo Rendering
    a. Possible to interconvert between the path space and the primary space.
    b. Compromise of the spaces.

References

1. *Metropolis-Hastings Algorithm*, Wikipedia, https://en.wikipedia.org/wiki/Metropolis-Hastings_algorithm
2. H. Yang, *Lecture 5*, KAIST CS423 lecture notes, https://github.com/hongseok-yang/probprog19/tree/master/Lectures/Lecture5
3. E. Veach and L. J. Guibas, *Metropolis Light Transport*, SIGGRAPH ‘97.
4. A. Benyoub, *Primary sample space and Multiplexed MLT*, 2014, http://anisb.github.io/slides.pdf
5. C. Kelemen and L. Szirmay-Kalos, *Simple and Robust Mutation Strategy for Metropolis Light Transport Algorithm*, Computer Graphics Forum. Vol. 21. No. 3. Oxford, UK: Blackwell Publishing, Inc, 2002.
6. T.-M. Li et al., *Anisotropic Gaussian Mutations for Metropolis Light Transport through Hessian-Hamiltonian Dynamics*, SIGGRAPH ‘15.
7. H. Otsu et al., *Fusing State Spaces for Markov Chain Monte Carlo Rendering*, ACM Trans. on Graphics, 2017.

Thank you!