# CS482: Radiometry and Rendering Equation

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(윤성의)

Course URL:
http://sglab.kaist.ac.kr/~sungeui/ICG/

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# Announcements

- Make a project team of 2 or 3 persons for your final project
  - Each student has a clear role
  - Declare the team at the noah board by Oct-5; you don't need to define the topic by then
- Each student
  - Present two papers related to the project
  - 20 min for each talk
- Each team
  - Give a mid-term review presentation for the project
  - Give the final project presentation

# Tentative schedule

10월

27일 Student Presentation 1
29일 Student Presentation 2

11월

3 Student Presentation 3
5 Student Presentation 4

12 Reserved

17 Mid. Project Presentation 1
19 Mid. Project Presentation 2

24 Student Presentation 1

12월

1 Student Presentation 2
3 Student Presentation 3

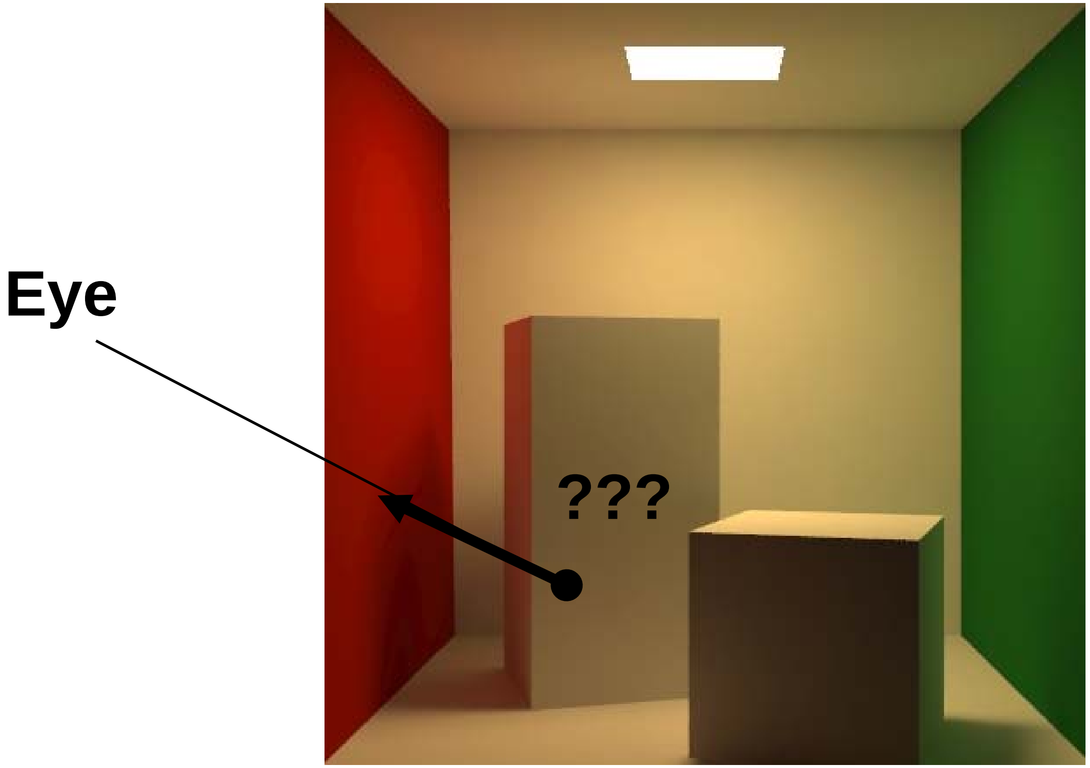
8 Student Presentation 4
10 Reserved

15 Final Presentation 1
17 Final Presentation 2

# Class Objectives

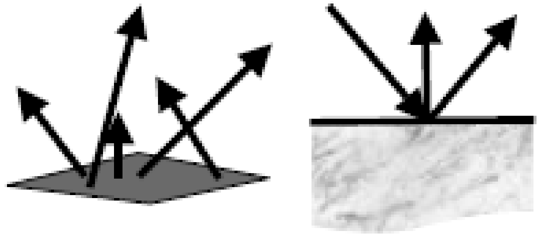
- **Know terms of:**
  - **Hemispherical coordinates and integration**
  - **Various radiometric quantities (e.g., radiance)**
  - **Basic material function, BRDF**
  - **Understand the rendering equation**

# Motivation



# Light and Material Interactions

- **Physics of light**



- **Radiometry**
- **Material properties**

From kavita's slides

- **Rendering equation**

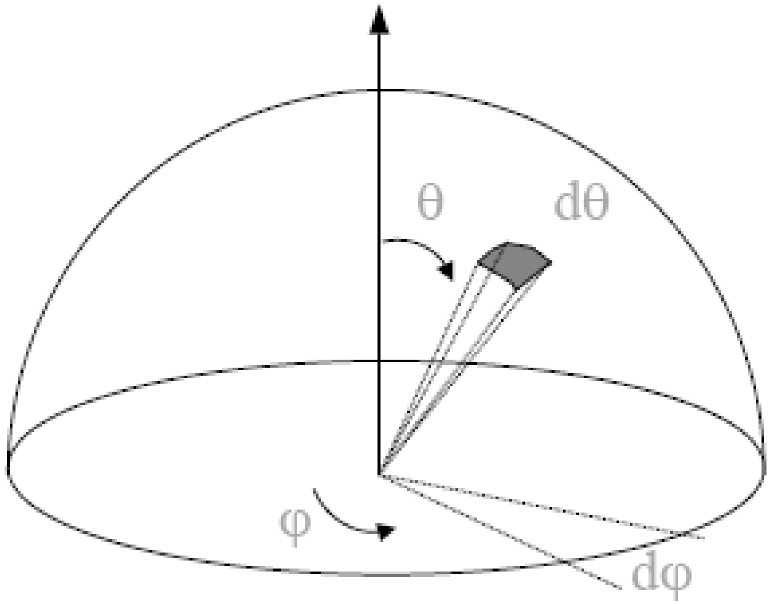
# Models of Light

- Quantum optics
  - Fundamental model of the light
  - Explain the dual wave-particle nature of light
- Wave model
  - Simplified quantum optics
  - Explains diffraction, interference, and polarization

- Geometric optics
  - Most commonly used model in CG
  - Size of objects >> wavelength of light
  - Light is emitted, reflected, and transmitted

# Hemispheres

- **Hemisphere**
  - **Two-dimensional surfaces**
- **Direction**
  - **Point on (unit) sphere**



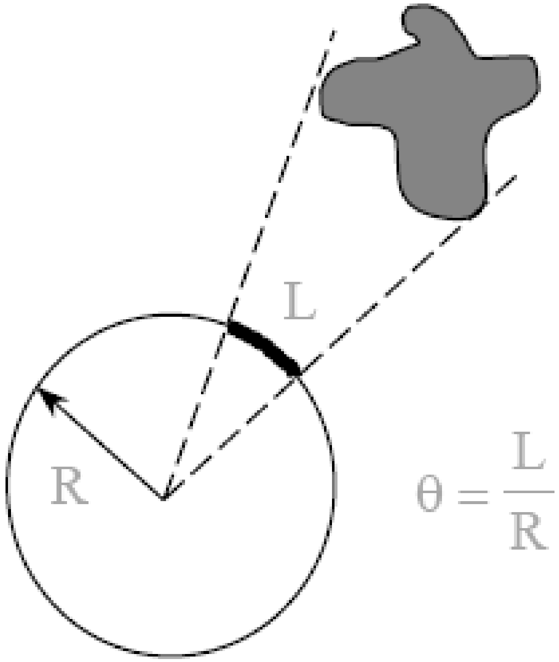

$$\theta \in [0, \frac{\pi}{2}]$$

$$\varphi \in [0, 2\pi]$$

From kavita's slides

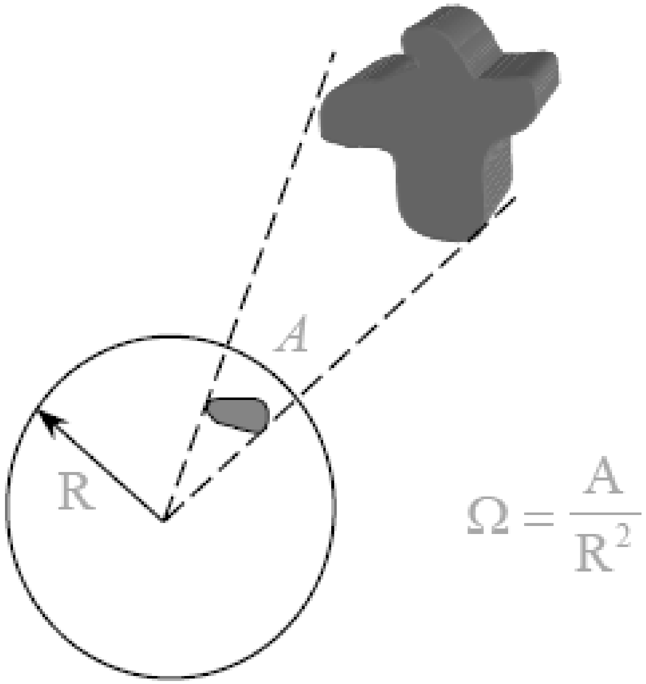
# Solid Angles

**2D**




**Full circle**
**= 2pi radians**

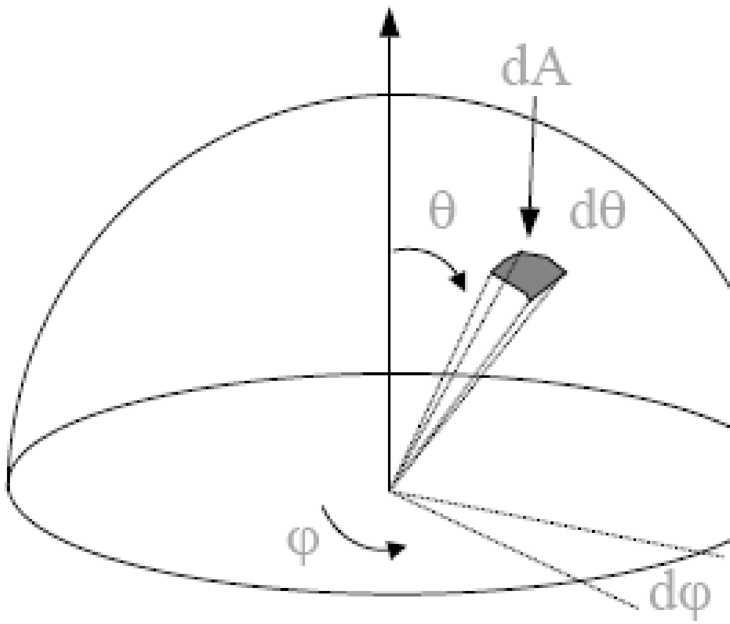
**3D**




**Full sphere**
**= 4pi steradians**

# Hemispherical Coordinates

- **Direction, $\Theta$**
  - **Point on (unit) sphere**




$$dA = (r \sin\theta d\varphi)(r d\theta)$$

From kavita's slides

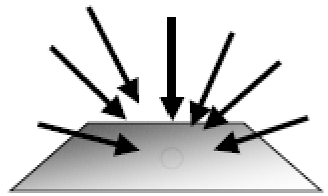
# Hemispherical Coordinates

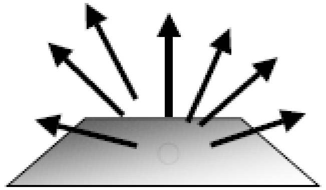
- **Differential solid angle**

$$d\omega = \frac{dA}{r^2} = \sin\theta d\theta d\varphi$$

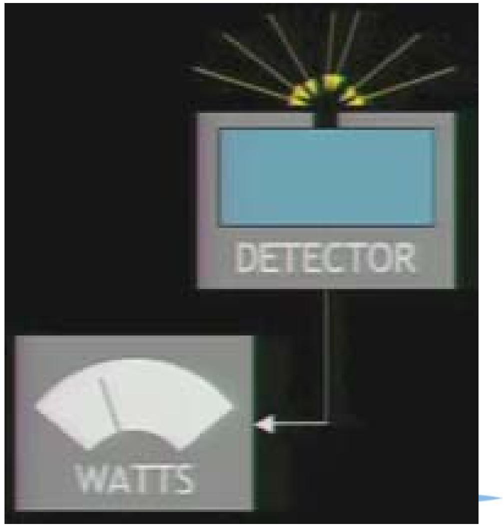
# Irradiance

- Incident radiant power per unit area (dP/dA)
  - Area density of power





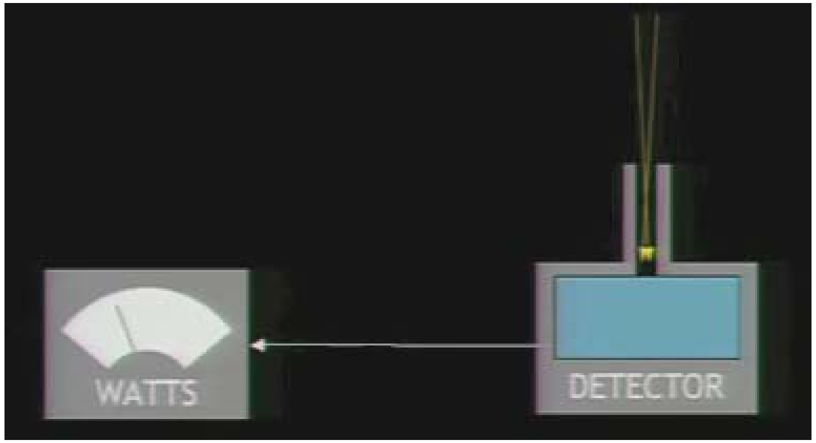
- Symbol: E, unit: W/ $m^2$
  - Area power density existing a surface is called radiance exitance (M) or radiosity (B)




- For example
  - A light source emitting 100 W of area 0.1 $m^2$
  - Its radiant exitance is 1000 W/ $m^2$

# Radiance

- **Radiant power at x in direction θ**
  - $L(x \rightarrow \Theta)$ **: 5D function**
    - **Per unit area**
    - **Per unit solid angle**

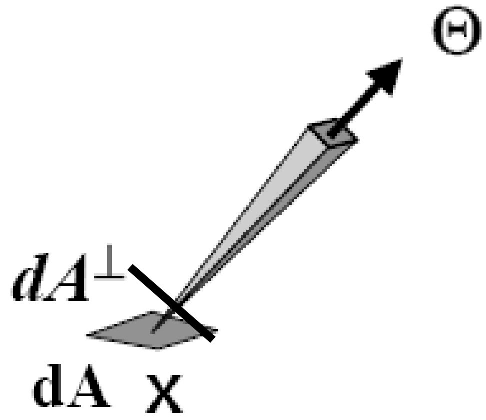



- **Important quantity for rendering**

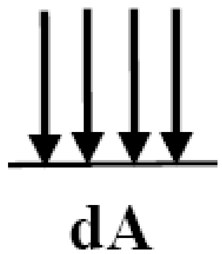
# Radiance: Projected Area

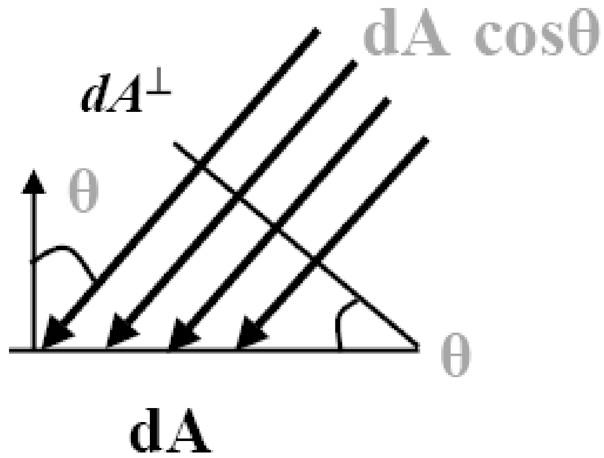
$$L(x \rightarrow \Theta) = \frac{d^2P}{dA^{\perp} d\omega_{\Theta}}$$

$$= \frac{d^2P}{d\omega_{\Theta}\, dA \cos\theta}$$



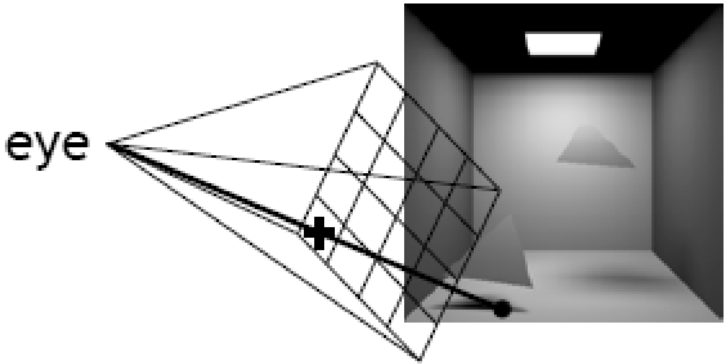

- **Why per unit projected surface area**



# Sensitivity to Radiance

- **Responses of sensors (camera, human eye) is proportional to radiance**

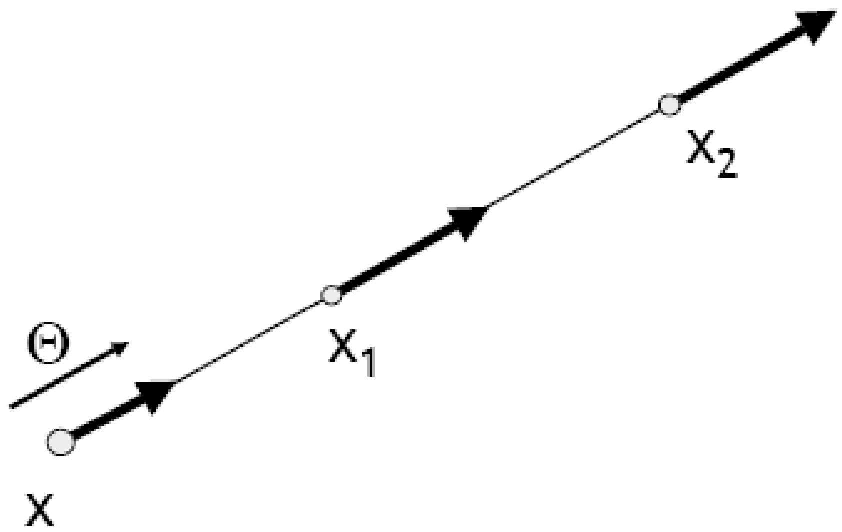



From kavita's slides

- **Pixel values in image proportional to radiance received from that direction**

# Properties of Radiance

- **Invariant along a straight line (in vacuum)**

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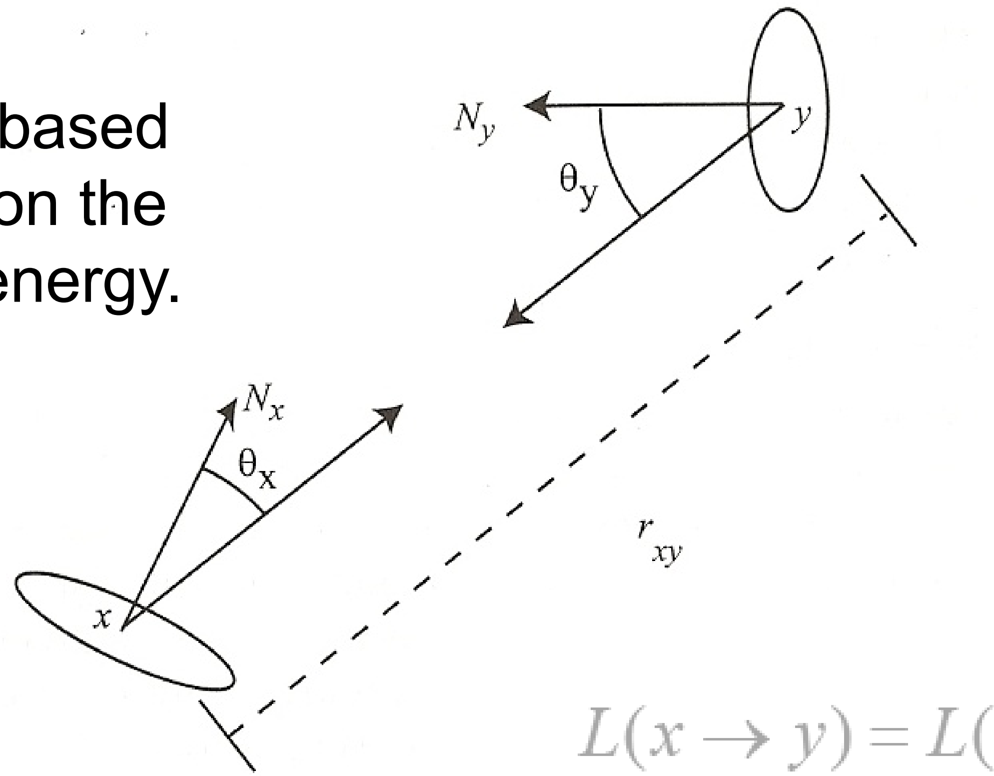
# Invariance of Radiance

We can prove it based on the assumption the conservation of energy.




$$L(x \rightarrow y) = L(y \leftarrow x)$$

Figure 2.3. Invariance of radiance.



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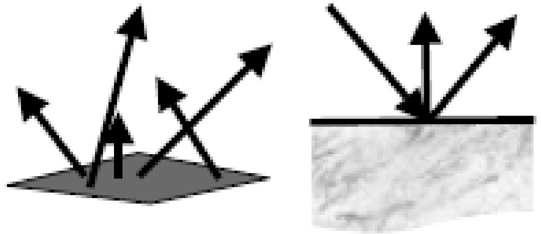
# Light and Material Interactions

- Physics of light



- Radiometry
- Material properties

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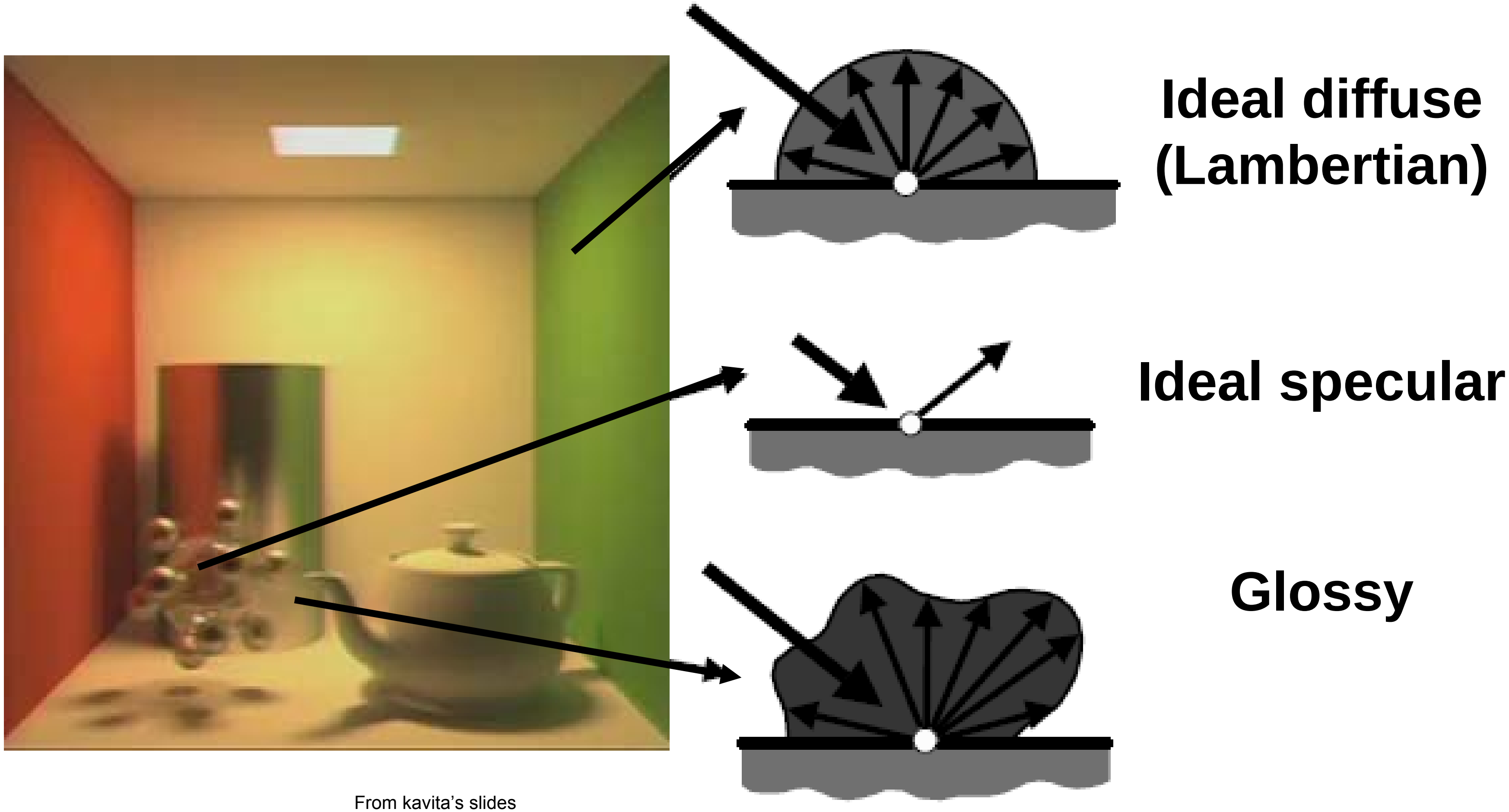
- Rendering equation

# Materials

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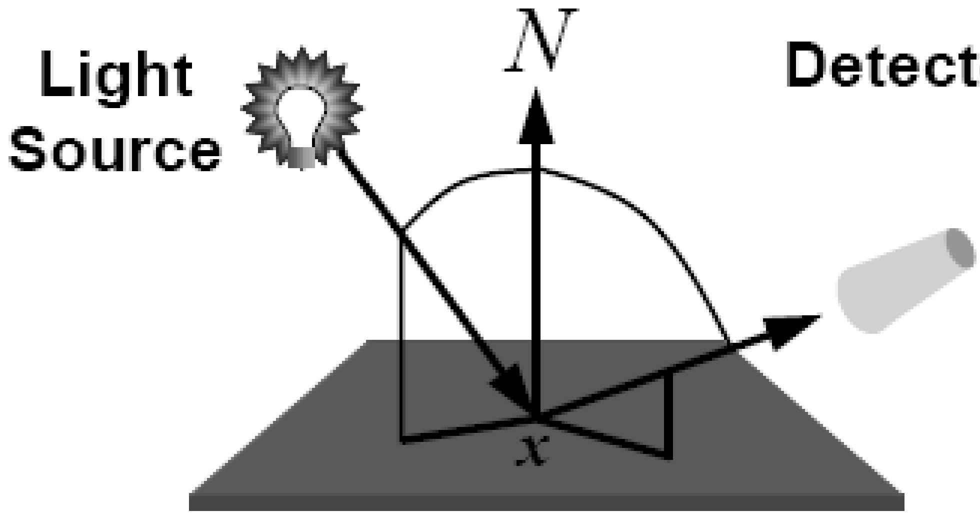
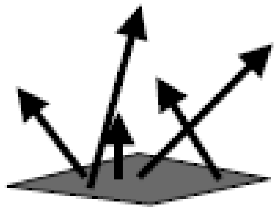
# Bidirectional Reflectance Distribution Function (BRDF)

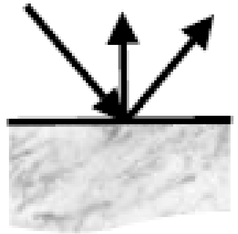



$$f_r(x, \Psi \to \Theta) = \frac{dL(x \to \Theta)}{dE(x \leftarrow \Psi)} = \frac{dL(x \to \Theta)}{L(x \leftarrow \Psi)\cos(N_x, \Psi)d\omega_\Psi}$$

# Light and Material Interactions

- **Physics of light**





- **Radiometry**

- **Material properties**

From kavita's slides

- **Rendering equation**

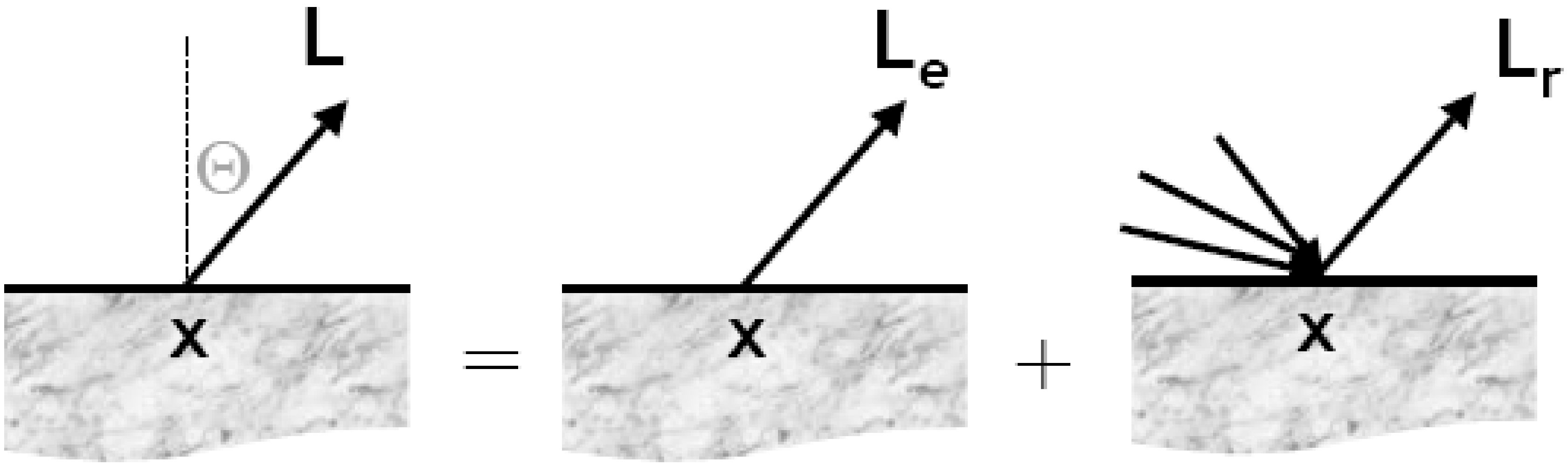
# Light Transport

- Goal
  - Describe steady-state radiance distribution in the scene
- Assumptions
  - Geometric optics
  - Achieves steady state instantaneously

# Rendering Equation

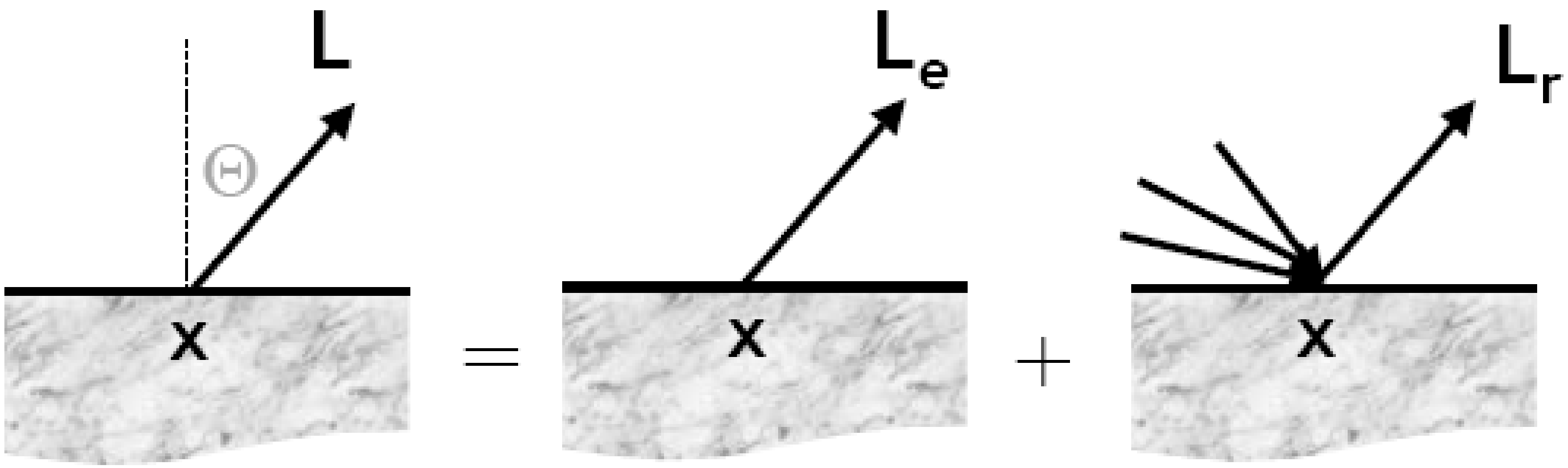
- Describes energy transport in the scene
- Input
  - Light sources
  - Surface geometry
  - Reflectance characteristics of surfaces
- Output
  - Value of radiances at all surface points in all directions

# Rendering Equation




$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + L_r(x \rightarrow \Theta)$$

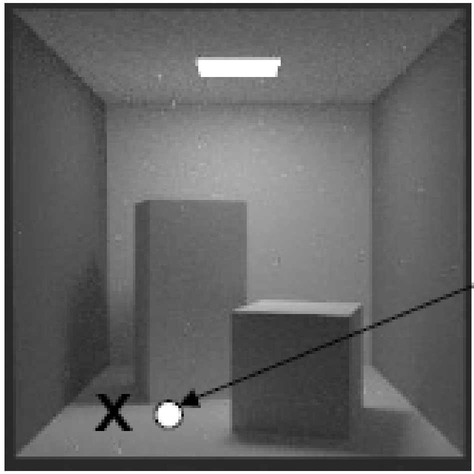
# Rendering Equation




$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{hemisphere} L(x \leftarrow \Psi)\, f_r(x, \Psi \leftrightarrow \Theta) \cos(\mathbf{N}_x, \Psi)\, d\omega_\Psi$$

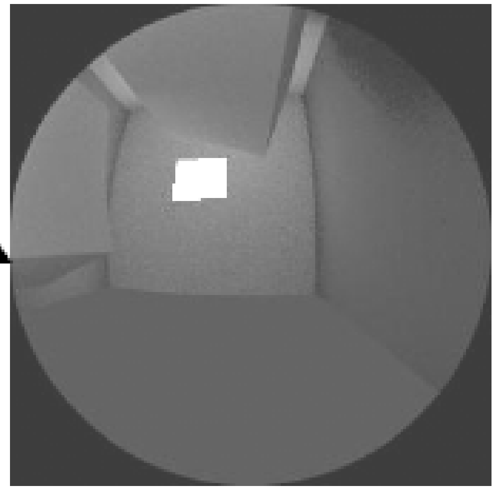
- Applicable for each wavelength

# Rendering Equation



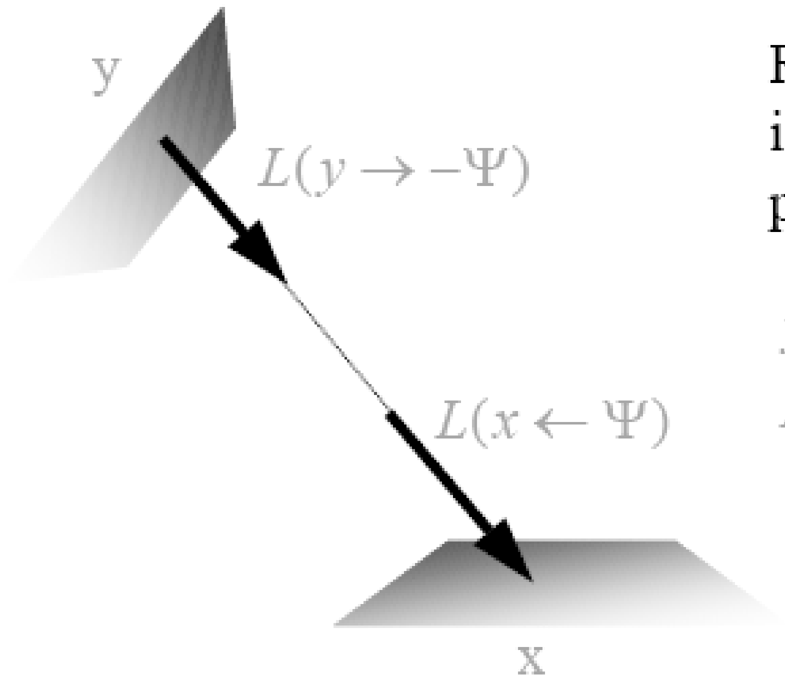

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{hemisphere} L(x \leftarrow \Psi) f_r(x, \Psi \leftrightarrow \Theta) \cos(\mathbf{N}_x, \Psi) d\omega_\Psi$$

incoming radiance



# Rendering Equation: Area Formulation

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$



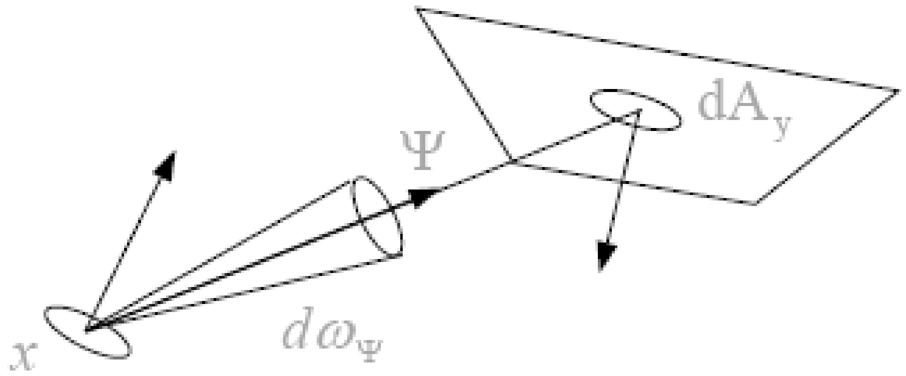

Ray-casting function: what is the nearest visible surface point seen from x in direction $\Psi$?

$$y = vp(x, \Psi)$$

$$L(x \leftarrow \Psi) = L(vp(x, \Psi) \rightarrow -\Psi)$$

# Rendering Equation

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$




$$y = vp(x, \Psi)$$

$$L(x \leftarrow \Psi) = L(vp(x, \Psi) \to -\Psi)$$

$$d\omega_\Psi = \frac{dA_y \cos\theta_y}{r_{xy}^2}$$

# Rendering Equation: Visible Surfaces

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$

Coordinate transform

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\substack{y \text{ on} \\ \text{all surfaces}}} f_r(\Psi \leftrightarrow \Theta) \cdot L(y \to -\Psi)\ \cos\theta_x \cdot \frac{\cos\theta_y}{r_{xy}^2} \cdot dA_y$$

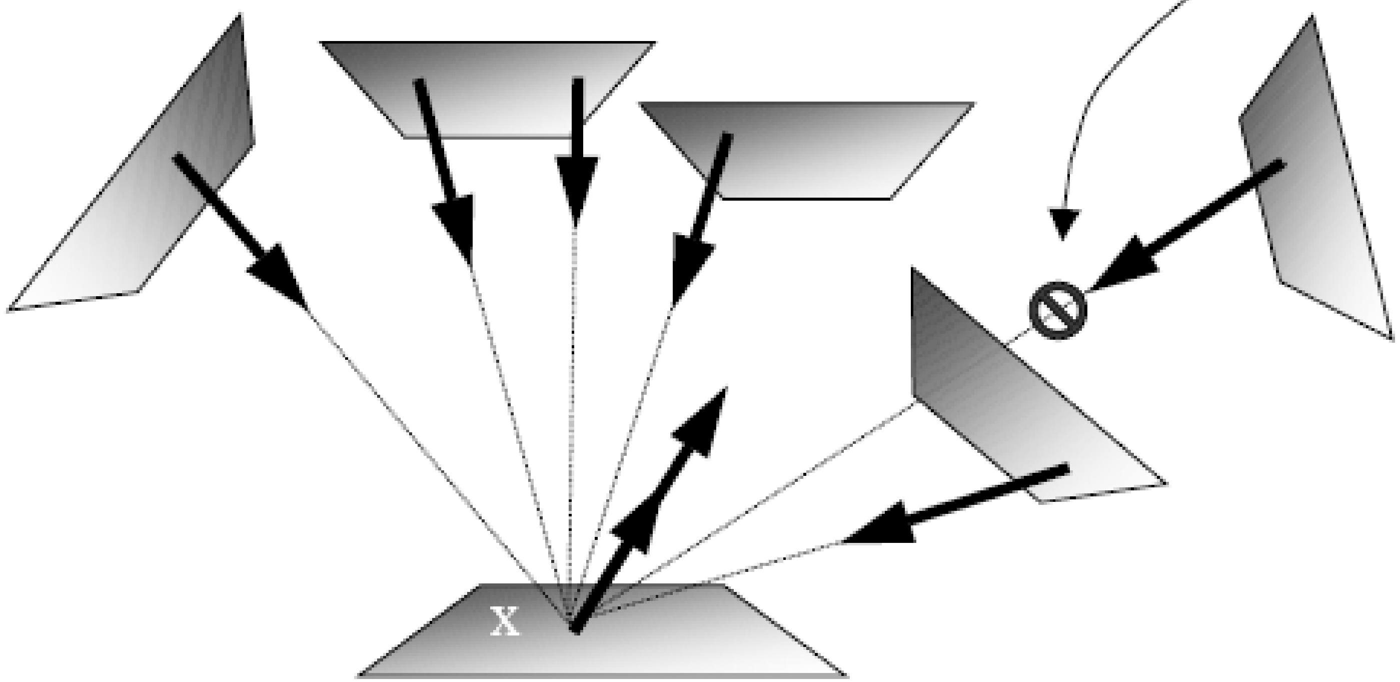
$$y = vp(x, \Psi)$$

Integration domain = visible surface points y

- Integration domain extended to ALL surface points by including visibility function

# Rendering Equation: All Surfaces

$$L(x \rightarrow \Theta) = L_e(\ldots) + \int_A f_r(\ldots) \cdot L(y \rightarrow -\Psi) \cdot \frac{\cos\theta_x \cdot \cos\theta_y}{r_{xy}^2} \cdot V(x,y)\, dA_y$$



# Two Forms of the Rendering Equation

- **Hemisphere integration**

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$

- **Area integration (used as the form factor)**

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_A f_r(\Psi \leftrightarrow \Theta) \cdot L(y \to -\Psi) \cdot \frac{\cos\theta_x \cdot \cos\theta_y}{r_{xy}^2} \cdot V(x,y) \cdot dA_y$$

# Class Objectives were:

- **Know terms of:**
  - **Hemispherical coordinates and integration**
  - **Various radiometric quantities (e.g., radiance)**
  - **Basic material function, BRDF**
  - **Understand the rendering equation**

# Next Time

- Monte Carlo rendering methods

# Homework

- **Go over the next lecture slides before the class**
- **Watch 2 SIG/I3D/HPG videos and submit your summaries every Tue. class**
  - **Just one paragraph for each summary**

**Example:**

**Title: XXX XXXX XXXX**

**Abstract: this video is about accelerating the performance of ray tracing. To achieve its goal, they design a new technique for reordering rays, since by doing so, they can improve the ray coherence and thus improve the overall performance.**

# Any Questions?

- Submit four times in Sep./Oct.
- Come up with one question on what we have discussed in the class and submit at the end of the class
  - 1 for typical questions
  - 2 for questions that have some thoughts or surprise me