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# CS686: RRT

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(윤성의)

**Course URL:**  
<http://sglab.kaist.ac.kr/~sungeui/MPA>

**KAIST**



# Class Objectives

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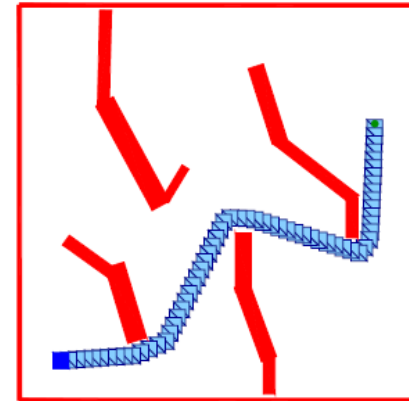
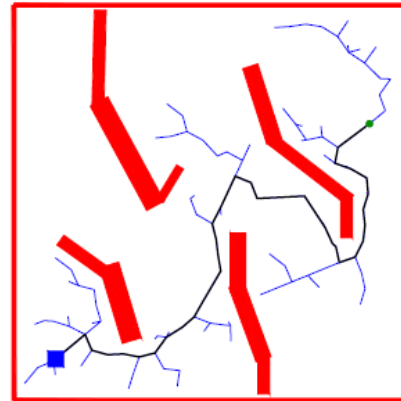
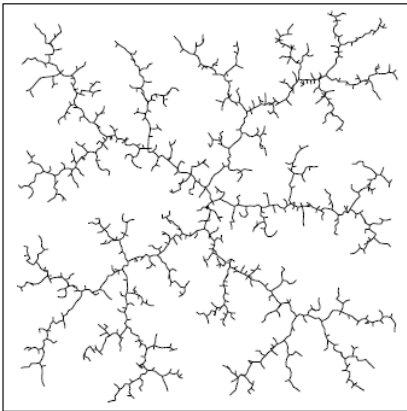
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- Understand the RRT technique and its recent advancements
  - RRT\*
  - Kinodynamic planning

# Rapidly-exploring Random Trees (RRT) [LaValle 98]

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- Present an efficient randomized path planning algorithm for single-query problems
  - Converges quickly
  - Probabilistically complete
  - Works well in high-dimensional C-space

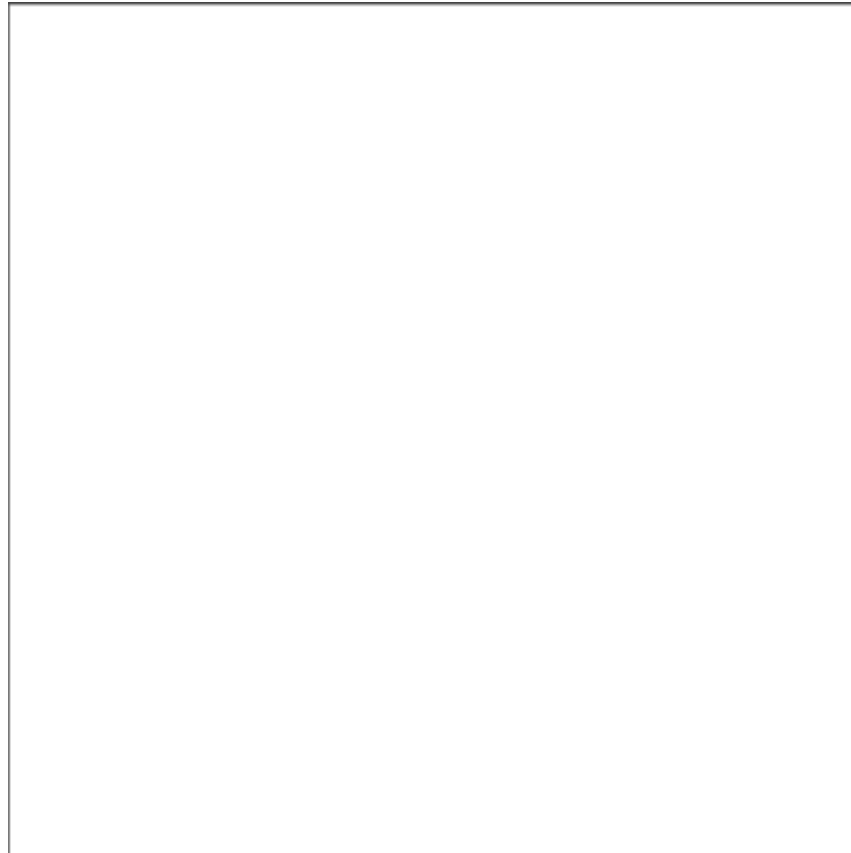


# Rapidly-Exploring Random Tree

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- A growing tree from an initial state

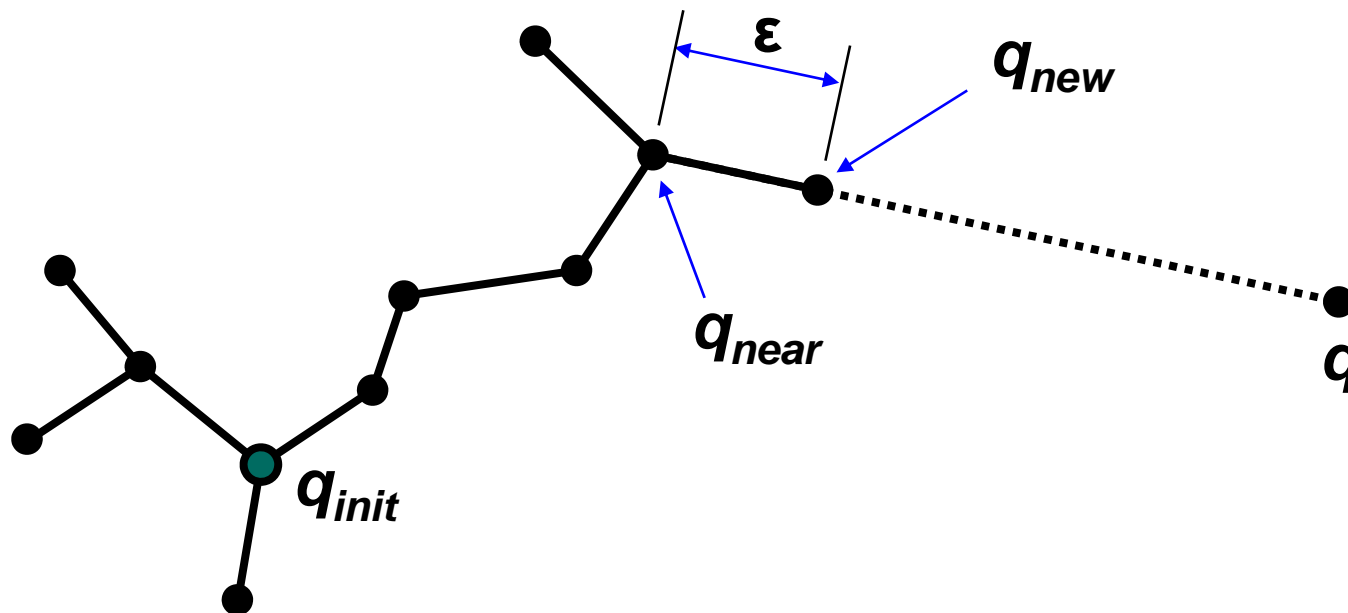


# RRT Construction Algorithm

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- Extend a new vertex in each iteration

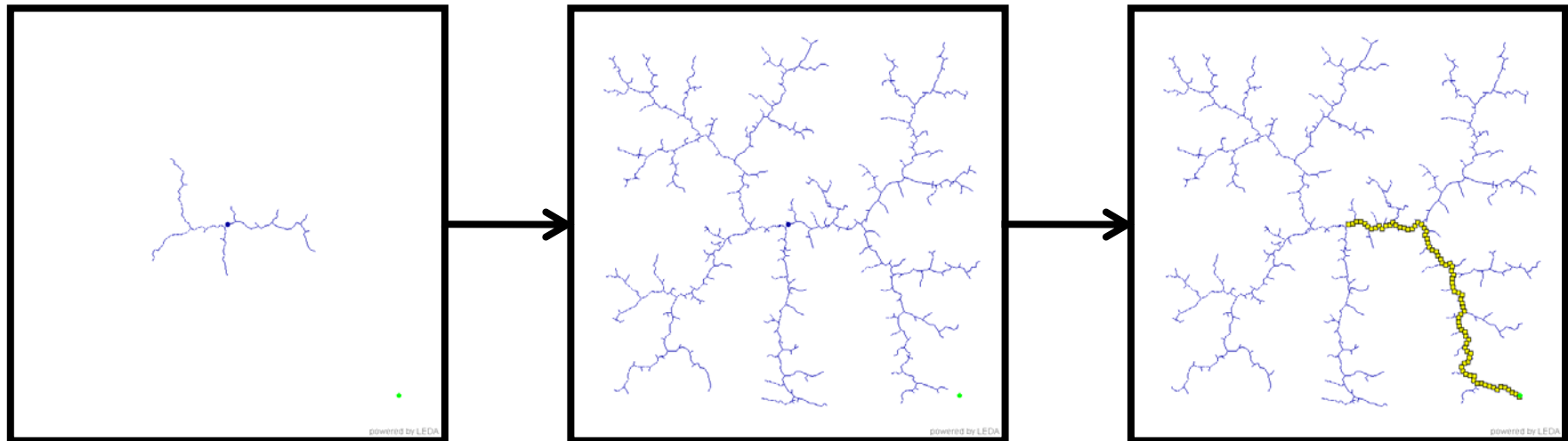


# Overview – Planning with RRT

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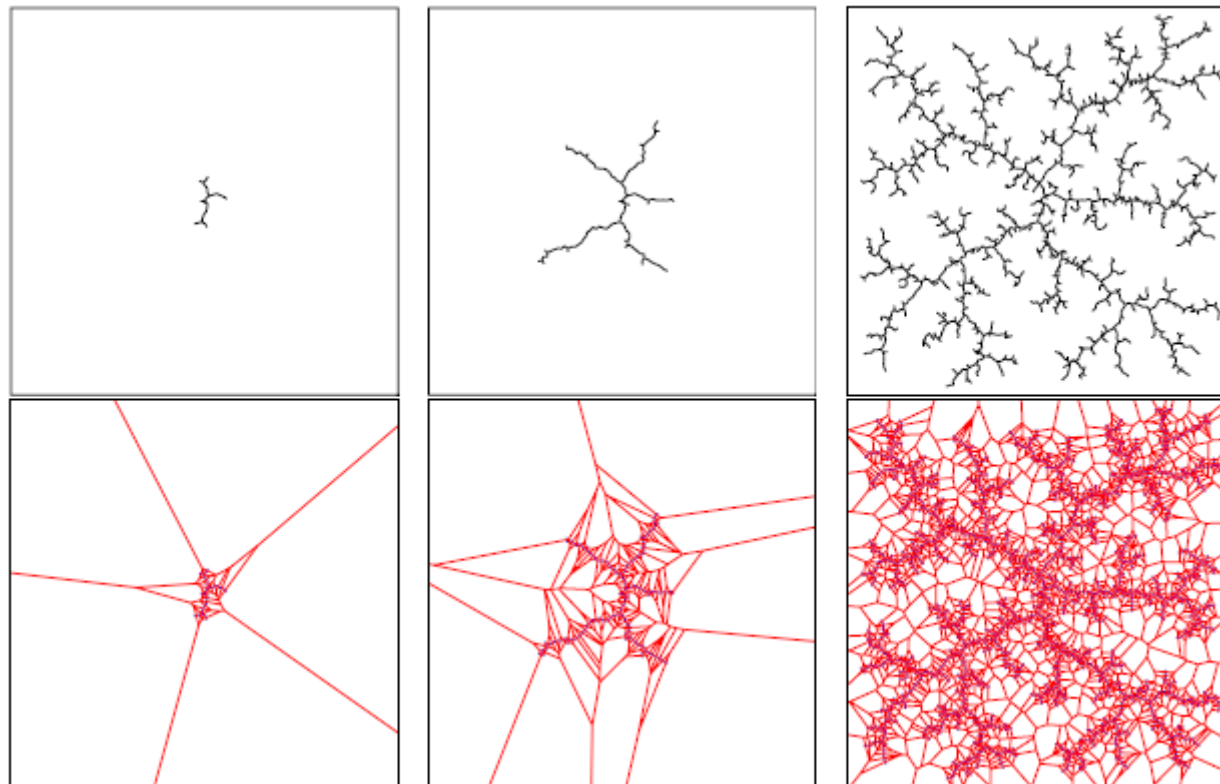
- **Extend RRT until a nearest vertex is close enough to the goal state**
  - Biased toward unexplored space
  - Can handle nonholonomic constraints and high degrees of freedom
- **Probabilistically complete, but does not converge**



# Voronoi Region

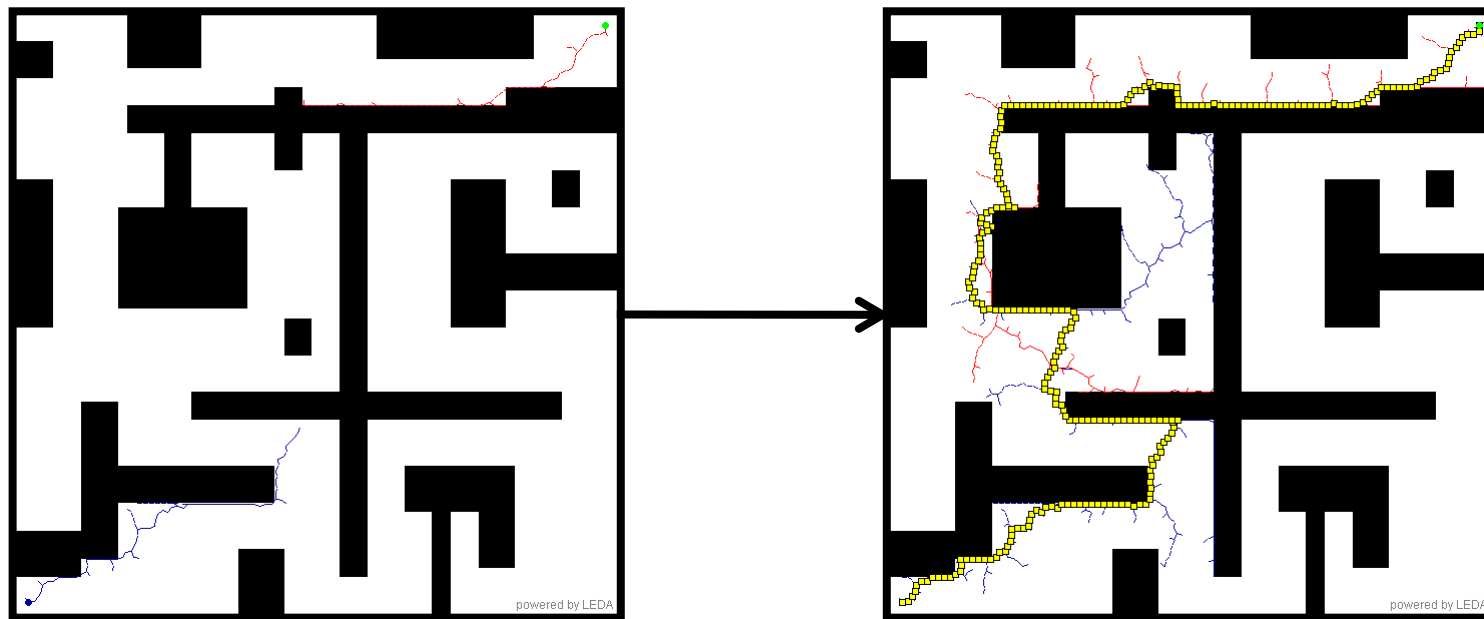
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- An RRT is biased by large Voronoi regions to rapidly explore, before uniformly covering the space



# Overview – With Dual RRT

- Extend RRTs from both initial and goal states
- Find path much more quickly

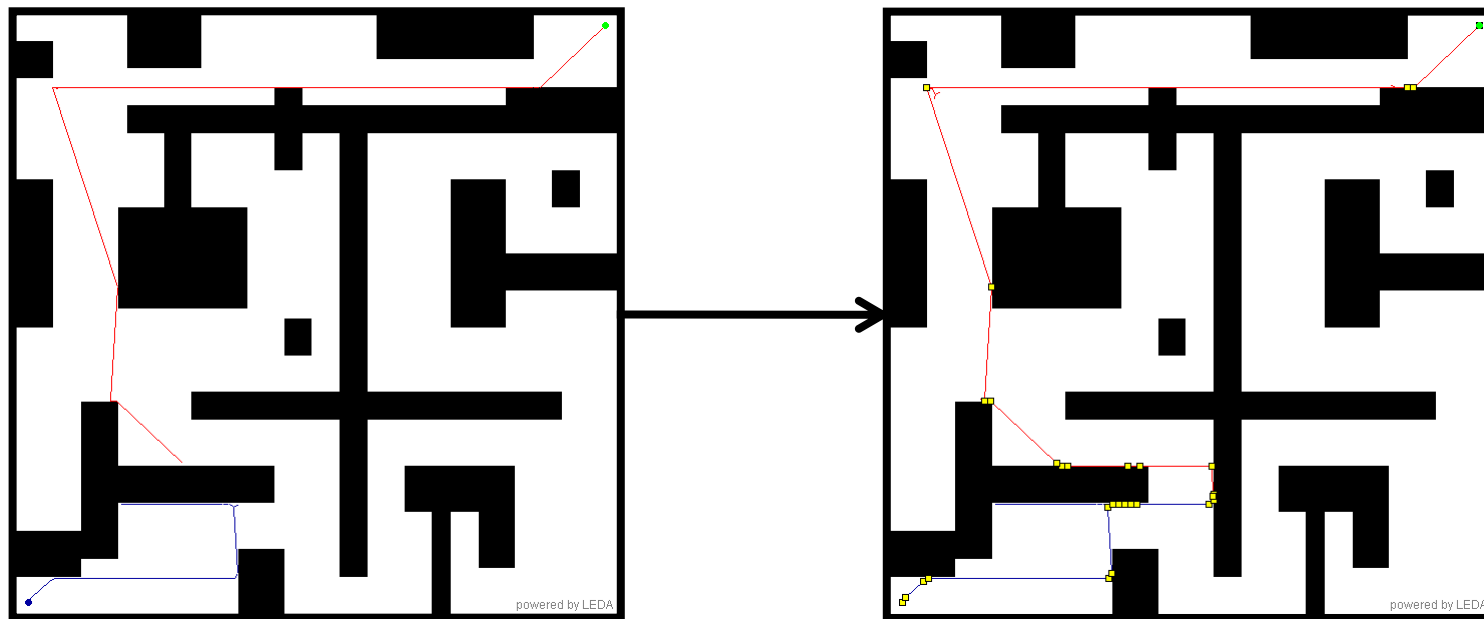


737 nodes are used



# Overview – With RRT-Connect

- Aggressively connect the dual trees using a greedy heuristic
- Extend & connect trees alternatively



42 nodes are used

# RRT Construction Algorithm

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```
BUILD_RRT( $q_{init}$ )
1   $\mathcal{T}.$ init( $q_{init}$ );
2  for  $k = 1$  to  $K$  do
3       $q_{rand} \leftarrow$  RANDOM_CONFIG();
4      EXTEND( $\mathcal{T}, q_{rand}$ );
5  Return  $\mathcal{T}$ 
```

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```
EXTEND( $\mathcal{T}, q$ )
1   $q_{near} \leftarrow$  NEAREST_NEIGHBOR( $q, \mathcal{T}$ );
2  if NEW_CONFIG( $q, q_{near}, q_{new}$ ) then
3       $\mathcal{T}.$ add_vertex( $q_{new}$ );
4       $\mathcal{T}.$ add_edge( $q_{near}, q_{new}$ );
5      if  $q_{new} = q$  then
6          Return Reached;
7      else
8          Return Advanced;
9  Return Trapped;
```

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# RRT Connect Algorithm

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CONNECT( $\mathcal{T}, q$ )

```
1  repeat
2     $S \leftarrow \text{EXTEND}(\mathcal{T}, q)$ ;
3  until not ( $S = \text{Advanced}$ )
4  Return  $S$ ;
```

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RRT\_CONNECT\_PLANNER( $q_{init}, q_{goal}$ )

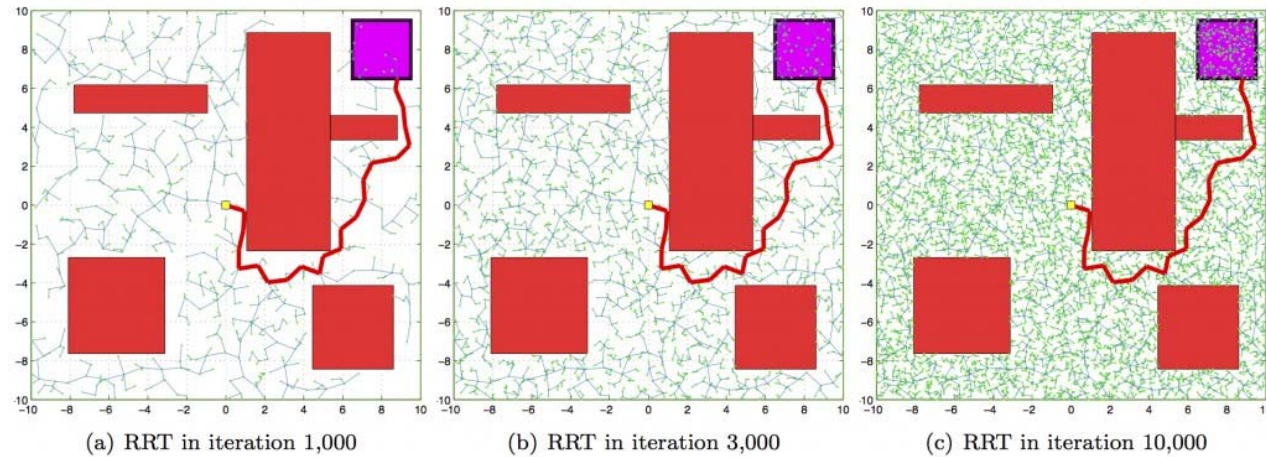
```
1   $\mathcal{T}_a.\text{init}(q_{init}); \mathcal{T}_b.\text{init}(q_{goal})$ ;
2  for  $k = 1$  to  $K$  do
3     $q_{rand} \leftarrow \text{RANDOM\_CONFIG}()$ ;
4    if not ( $\text{EXTEND}(\mathcal{T}_a, q_{rand}) = \text{Trapped}$ ) then
5      if ( $\text{CONNECT}(\mathcal{T}_b, q_{new}) = \text{Reached}$ ) then
6        Return  $\text{PATH}(\mathcal{T}_a, \mathcal{T}_b)$ ;
7    SWAP( $\mathcal{T}_a, \mathcal{T}_b$ );
8  Return Failure
```

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# RRT\*

- RRT does not converge to the optimal solution

RRT



RRT\*

# RRT\*

- **Asymptotically optimal without a substantial computational overhead**

**Theorem [Karaman & Frazzoli, IJRR 2011]**

(i) The RRT\* algorithm is asymptotically optimal

$$\mathbb{P}\left(\left\{\lim_{n \rightarrow \infty} Y_n^{\text{RRT}^*} = c^*\right\}\right) = 1$$

(ii) RRT\* algorithm has no substantial computational overhead when compared to the RRT:

$$\lim_{n \rightarrow \infty} \mathbb{E} \left[ \frac{M_n^{\text{RRT}^*}}{M_n^{\text{RRT}}} \right] = \text{constant}$$

- $Y_n^{\text{RRT}^*}$  : cost of the best path in the RRT\*
- $c^*$  : cost of an optimal solution
- $M_n^{\text{RRT}}$  : # of steps executed by RRT at iteration n
- $M_n^{\text{RRT}^*}$  : # of steps executed by RRT\* at iteration n

# Key Operation of RRT\*

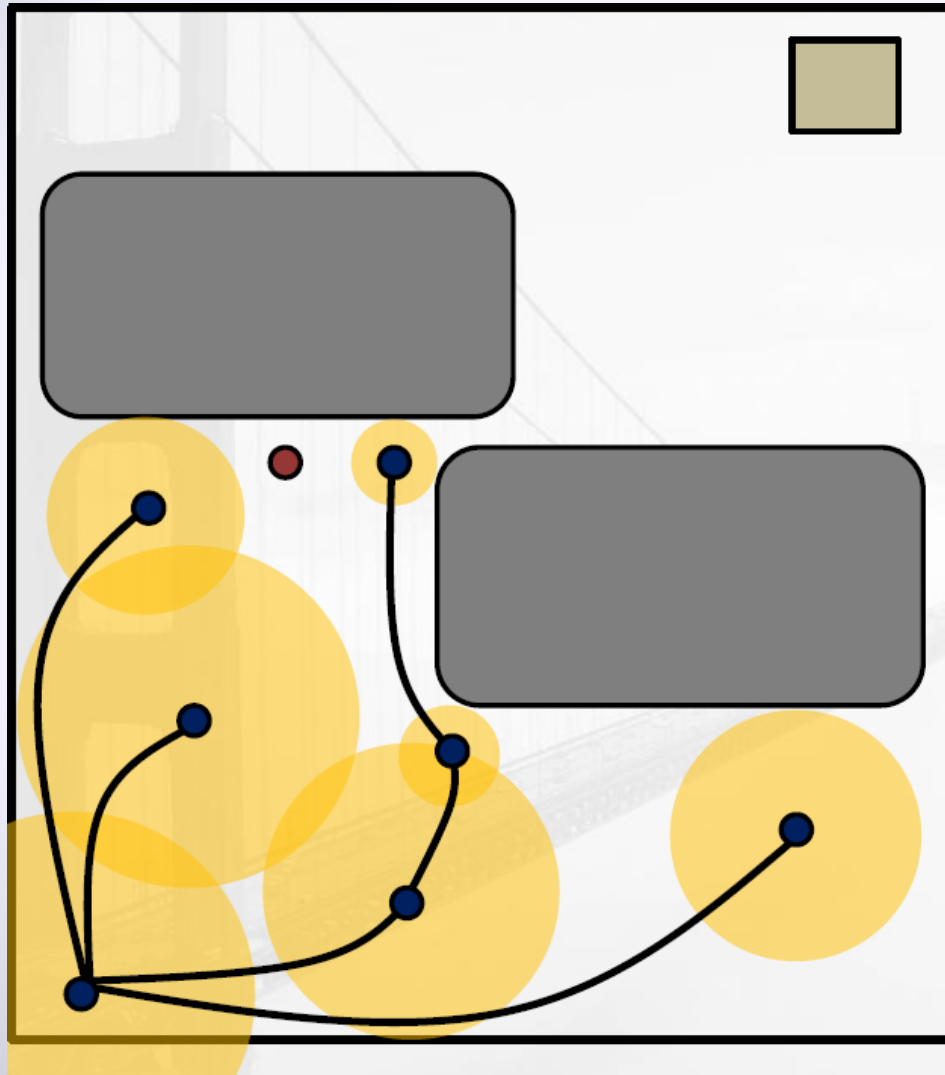
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- **RRT**
  - Just connect a new node to its nearest neighbor node
- **RRT\* : refine the connection with re-wiring operation**
  - Given a ball, identify neighbor nodes to the new node
  - Refine the connection to have a lower cost



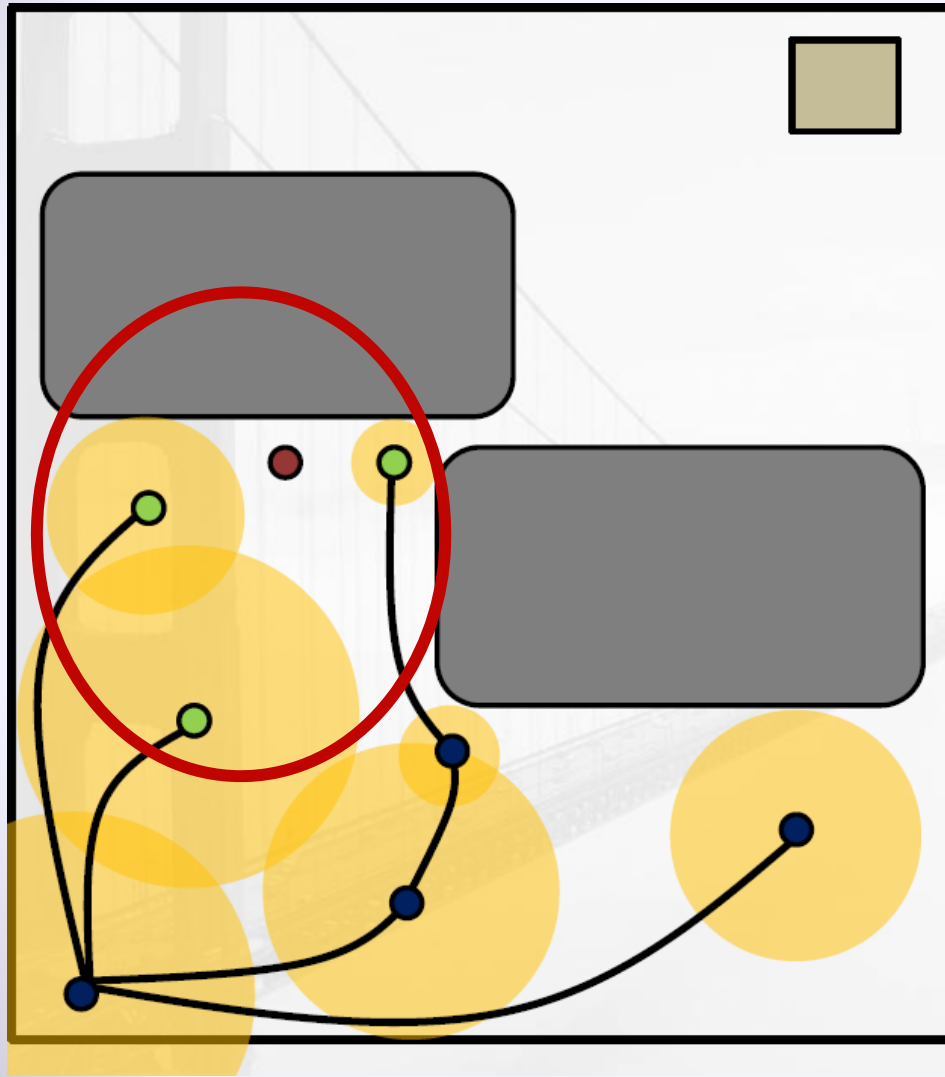
# Example: Re-Wiring Operation



Generate a new sample

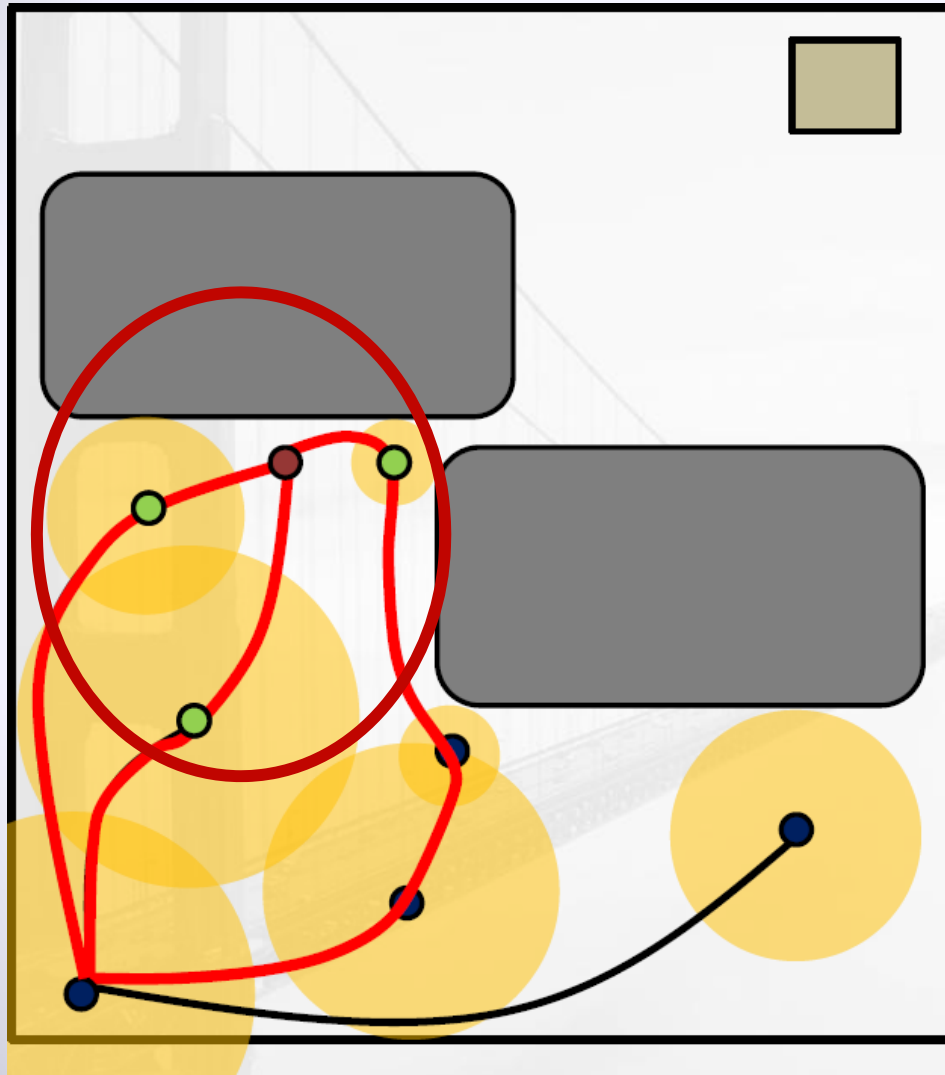


# Example: Re-Wiring Operation



Identify nodes in a ball

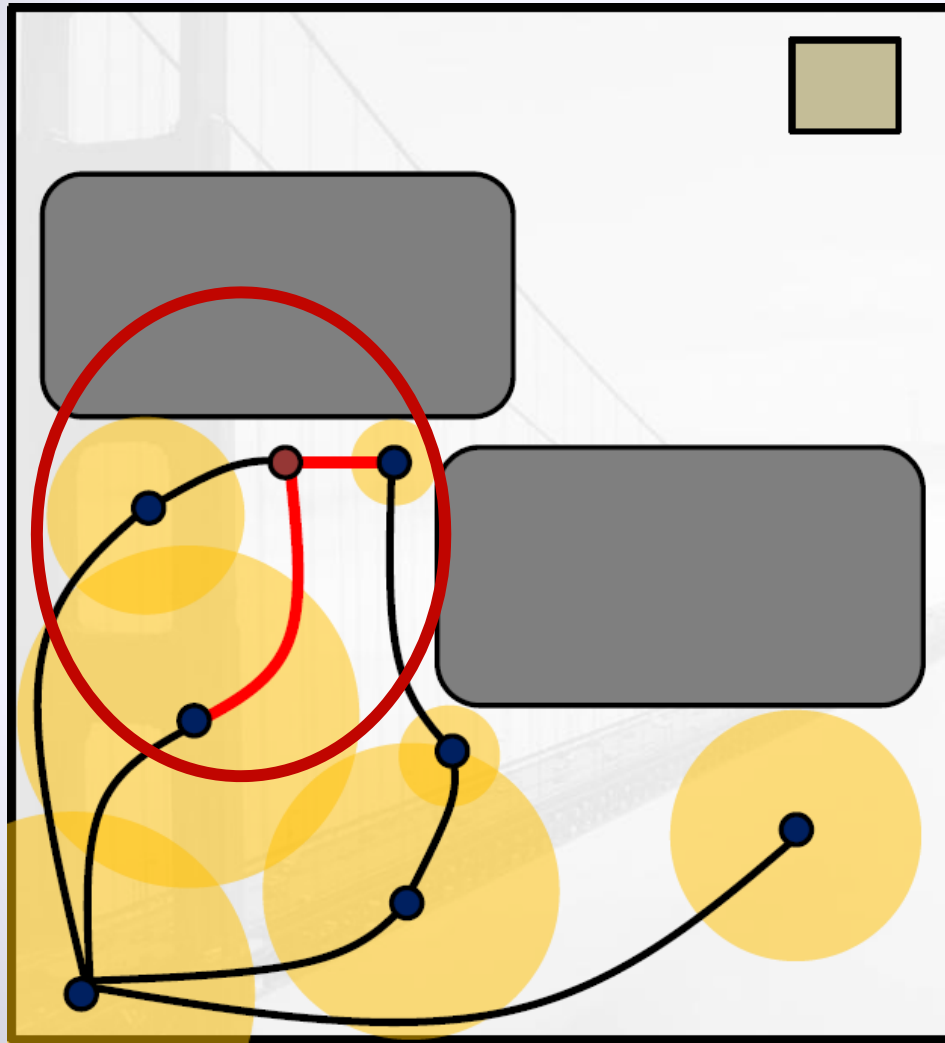
# Example: Re-Wiring Operation



Identify which parent gives the lowest cost

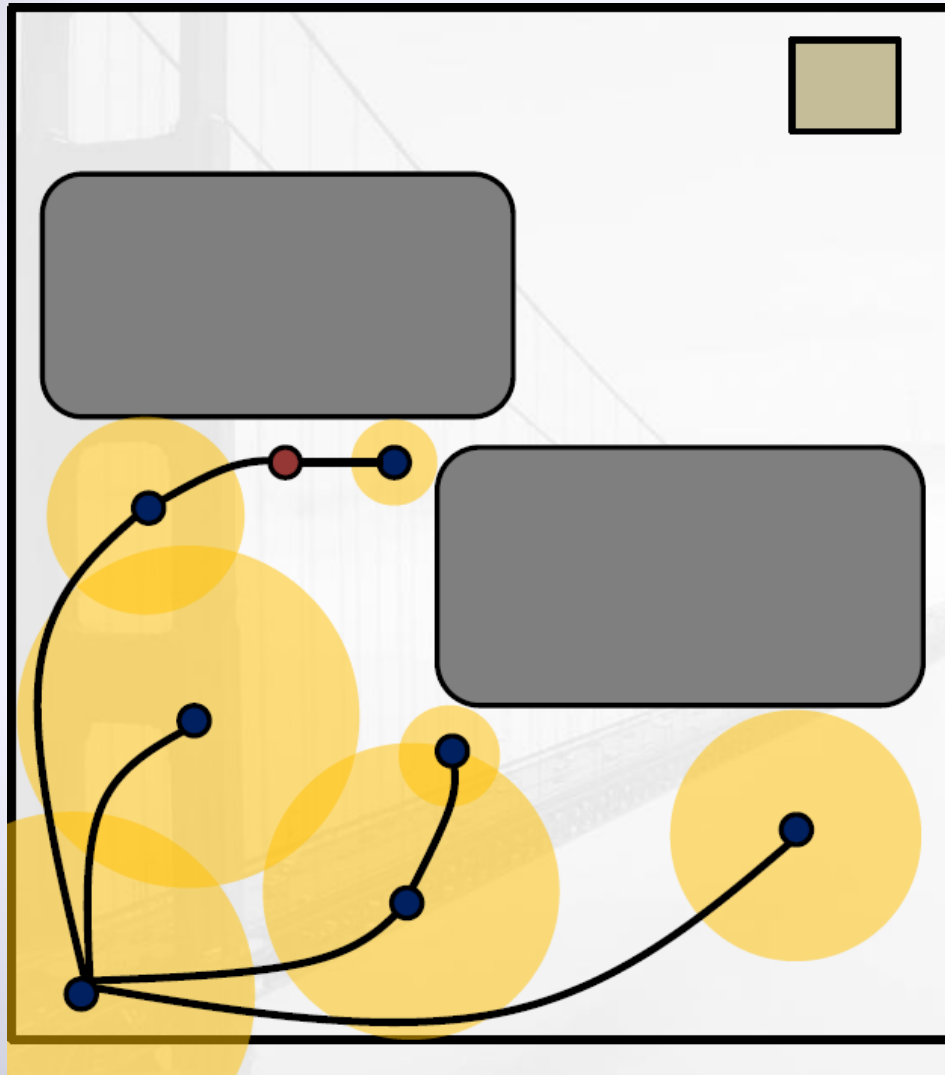


# Example: Re-Wiring Operation



Identify which child  
gives the lowest cost

# Example: Re-Wiring Operation



Video showing benefits  
with real robot

# Kinodynamic Path Planning

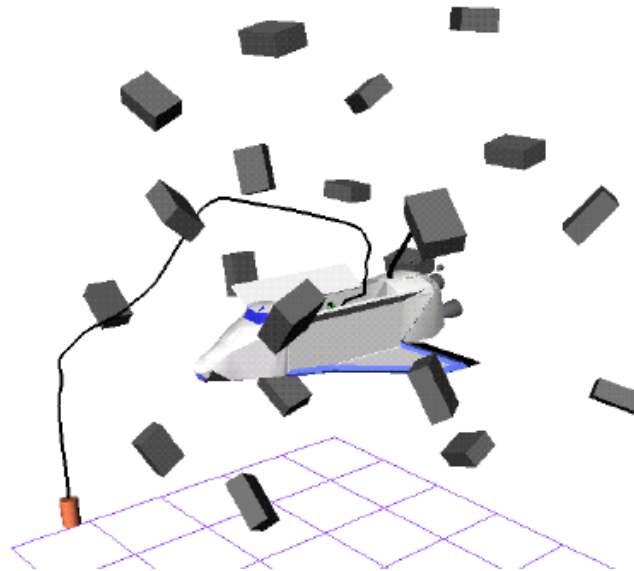
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ALSO GIVEN:  $h_i(q, \dot{q}, \ddot{q}) \leq 0, h_i(q, \dot{q}, \ddot{q}) = 0, \dots$

FIND:  $\tau$  that satisfies  $f_i(q), g_i(q, \dot{q}), h_i(q, \dot{q}, \ddot{q})$

- Consider kinematic + dynamic constraints



# State Space Formulation

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- **Kinodynamic planning → 2n-dimensional state space**

$C$  denote the  $C$ -space

$X$  denote the state space

$$x = (q, \dot{q}), \text{ for } q \in C, x \in X$$

$$x = [q_1 \quad q_2 \quad \dots \quad q_n \quad \frac{dq_1}{dt} \quad \frac{dq_2}{dt} \quad \dots \quad \frac{dq_n}{dt}]$$

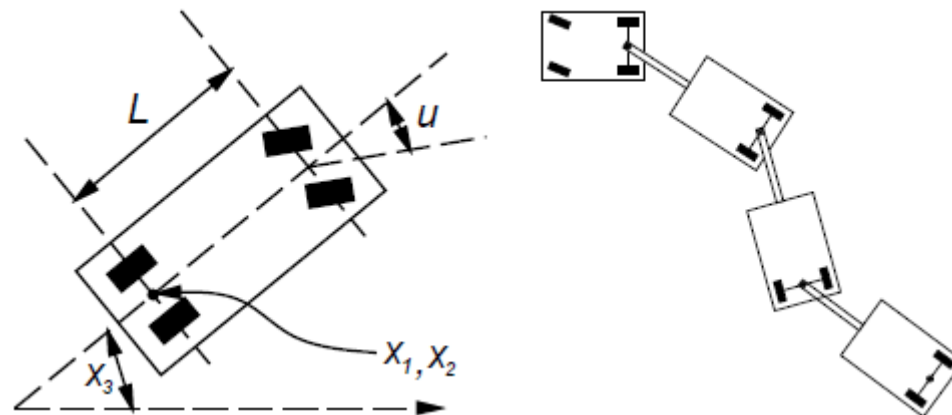
# Constraints in State Space

$h_i(q, \dot{q}, \ddot{q}) = 0$  becomes  $G_i(x, \dot{x}) = 0$ ,  
for  $i = 1, \dots, m$  and  $m < 2n$

- Constraints can be written in:

$$\dot{x} = f(x, u)$$

$u \in U$ ,  $U$  : Set of allowable controls or inputs





# Solution Trajectory

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- **Defined as a time-parameterized continuous path**

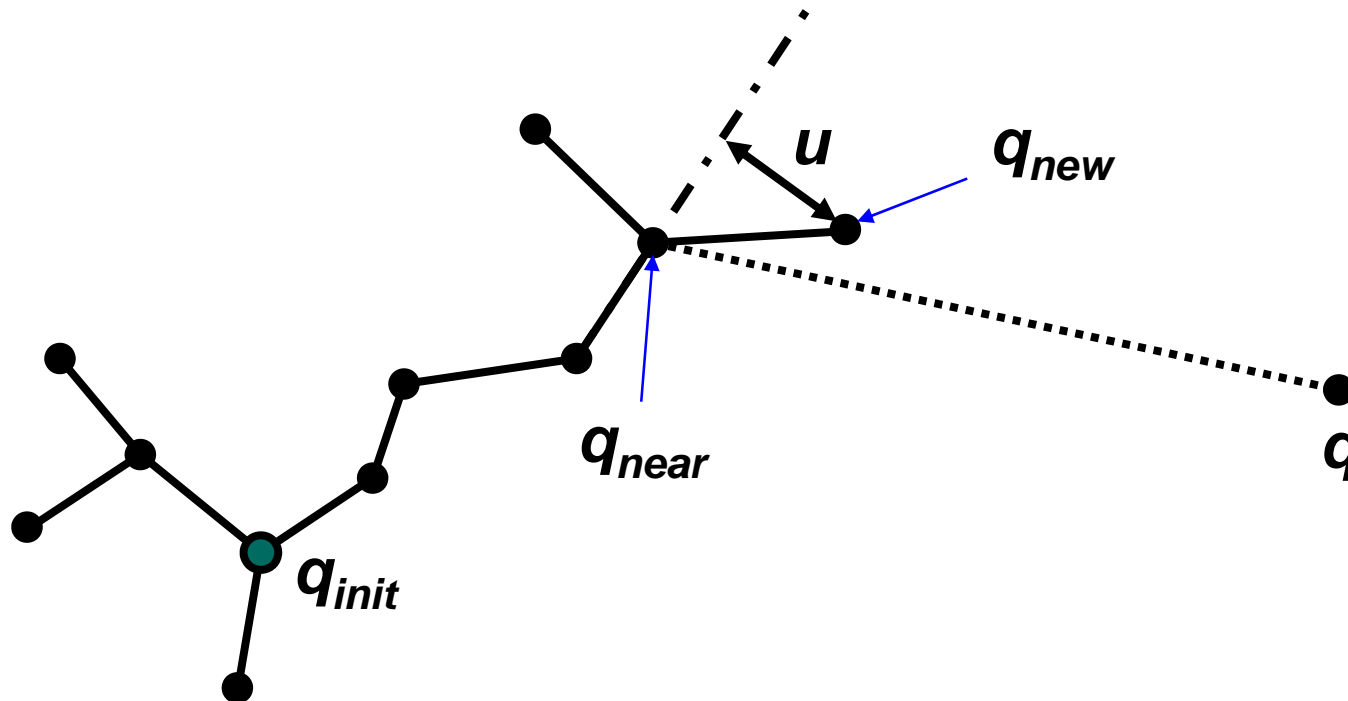
$\tau : [0, T] \rightarrow X_{free}$ , satisfies the constraints

- **Obtained by integrating  $\dot{x} = f(x, u)$**
- **Solution: Finding a control function**

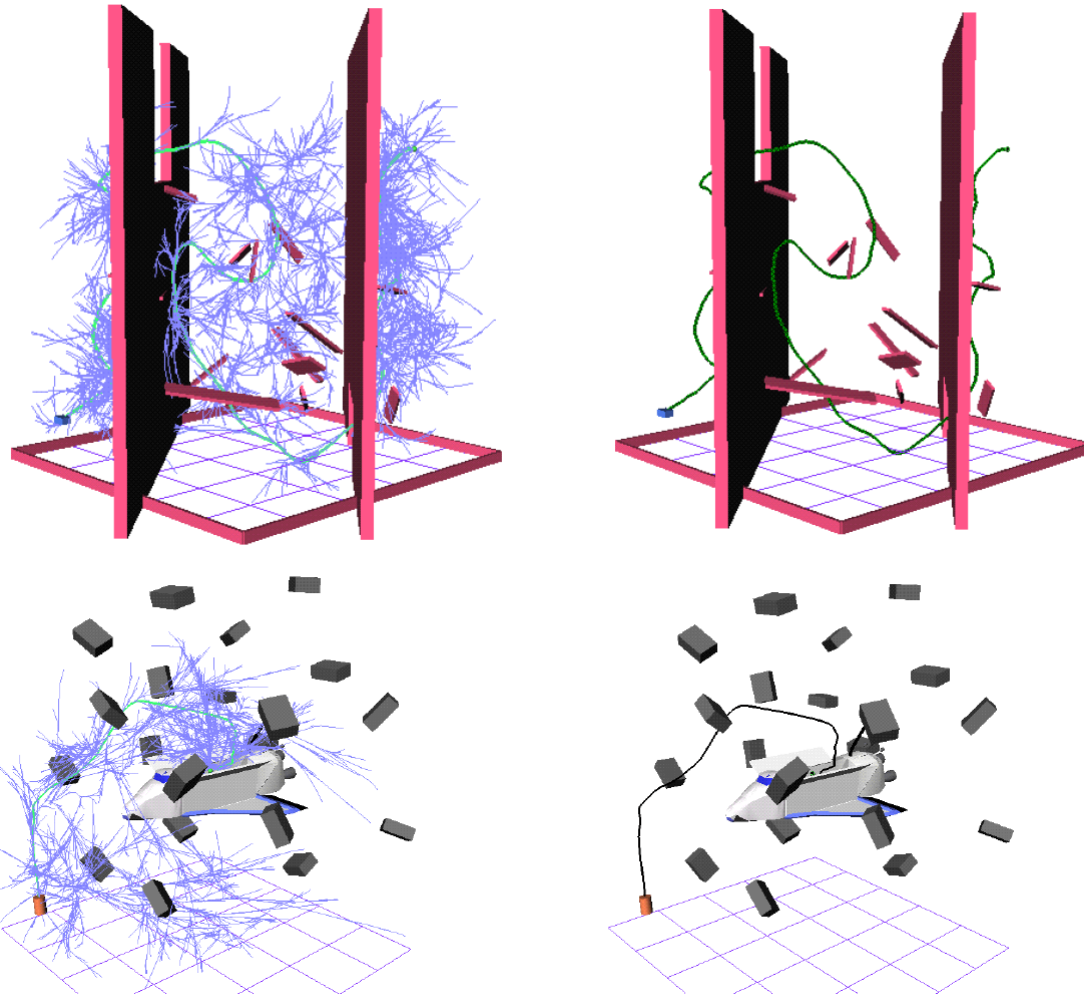
$$u : [0, T] \rightarrow U$$

# Rapidly-Exploring Random Tree

- Extend a new vertex in each iteration



# Results – 200MHz, 128MB



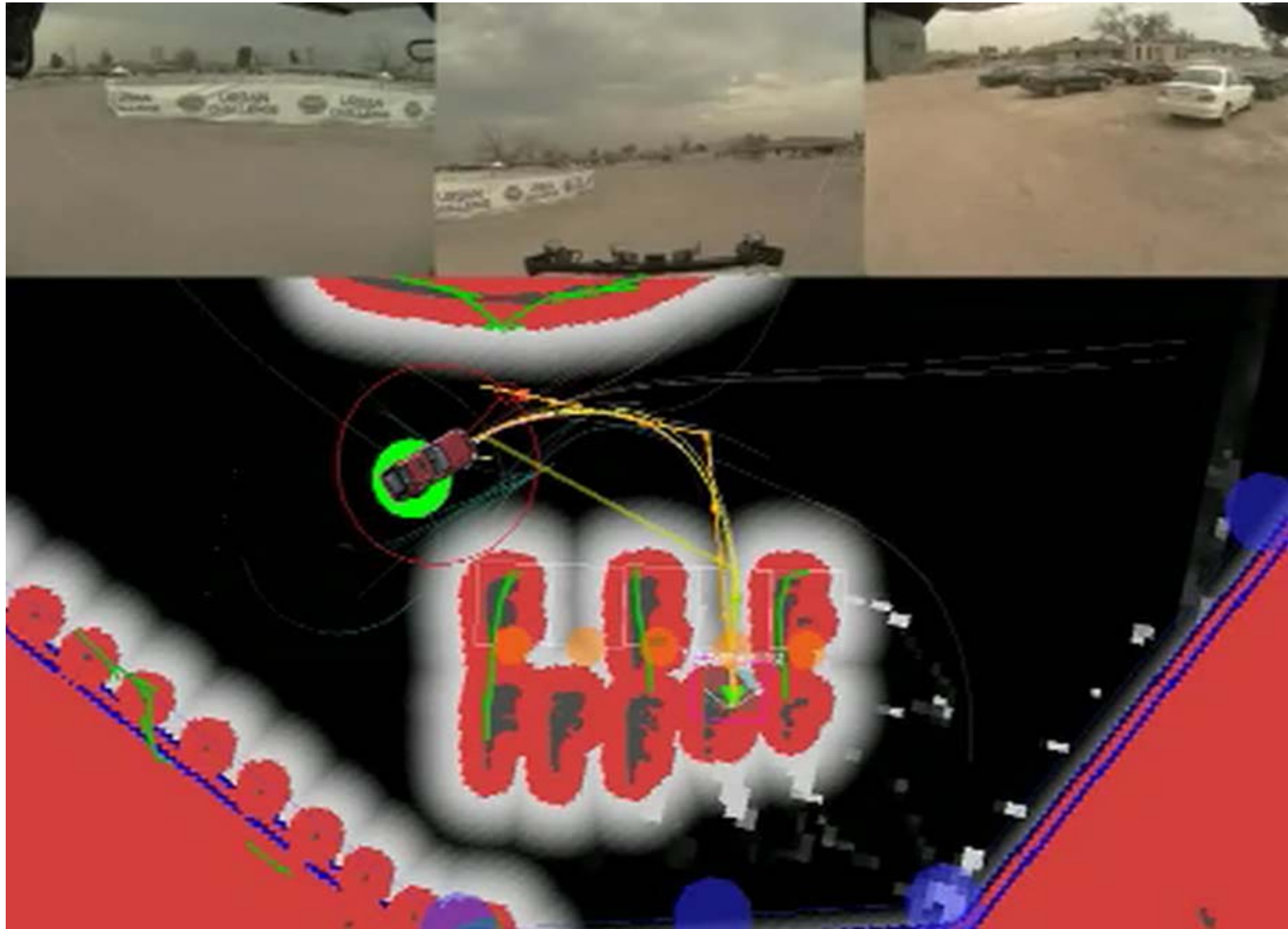
- 3D translating
- X=6 DOF
- 16,300 nodes
- 4.1min
  
- 3D TR+RO
- X=12 DOF
- 23,800 nodes
- 8.4min

# RRT at work: Urban Challenge

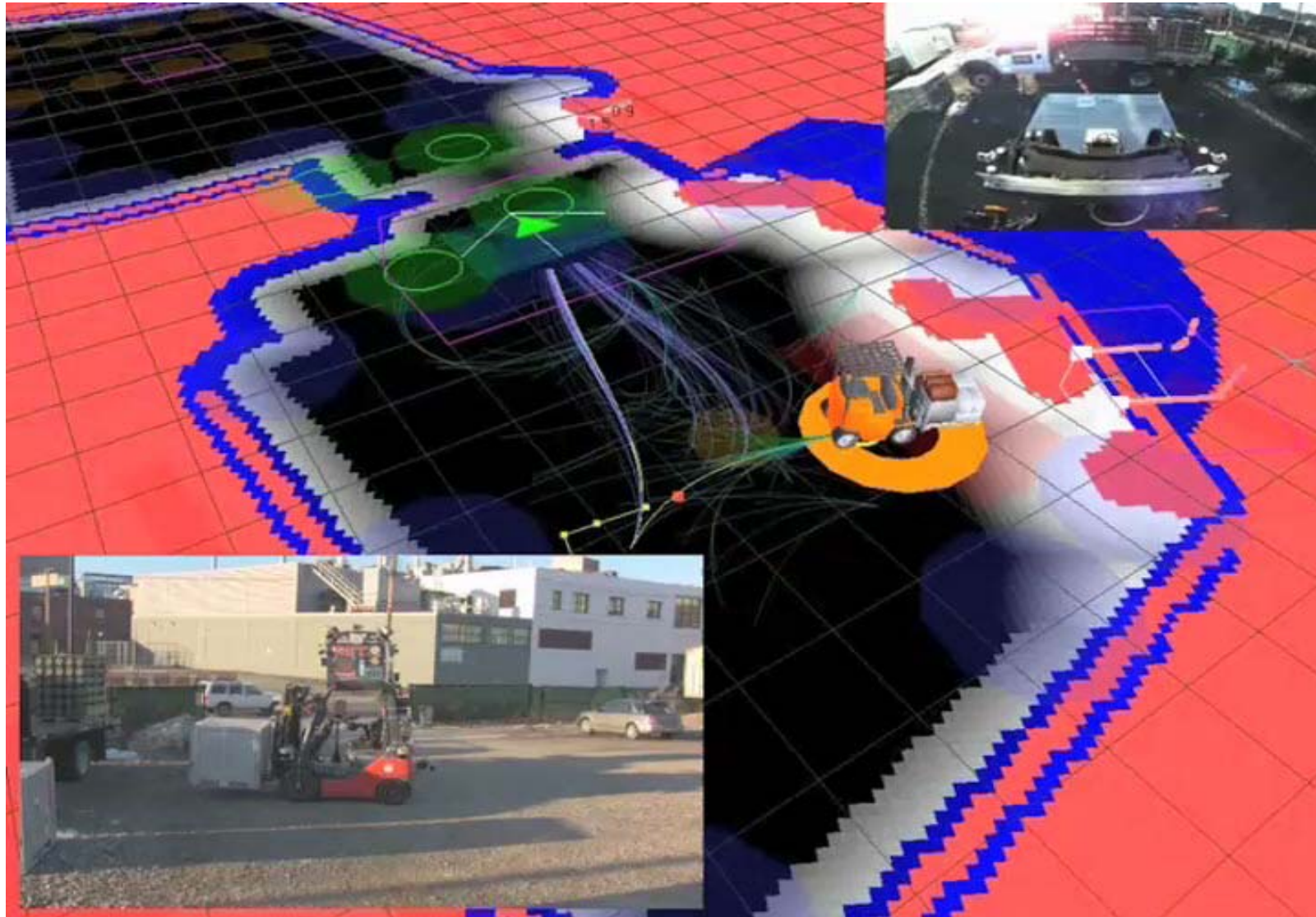


From MIT

# Successful Parking Maneuver



# RRT at work: Autonomous Forklift



# Recent Works of Our Group

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- **Narrow passages**
  - Identify narrow passage with a simple one-dimensional line test, and selectively explore such regions
  - Selective retraction-based RRT planner for various environments, Lee et al., T-RO 14
  - <http://sglab.kaist.ac.kr/SRRRT/T-RO.html>

# Retraction-based RRT

## [Zhang & Manocha 08]

- Retraction-based RRT technique **handling narrow passages**

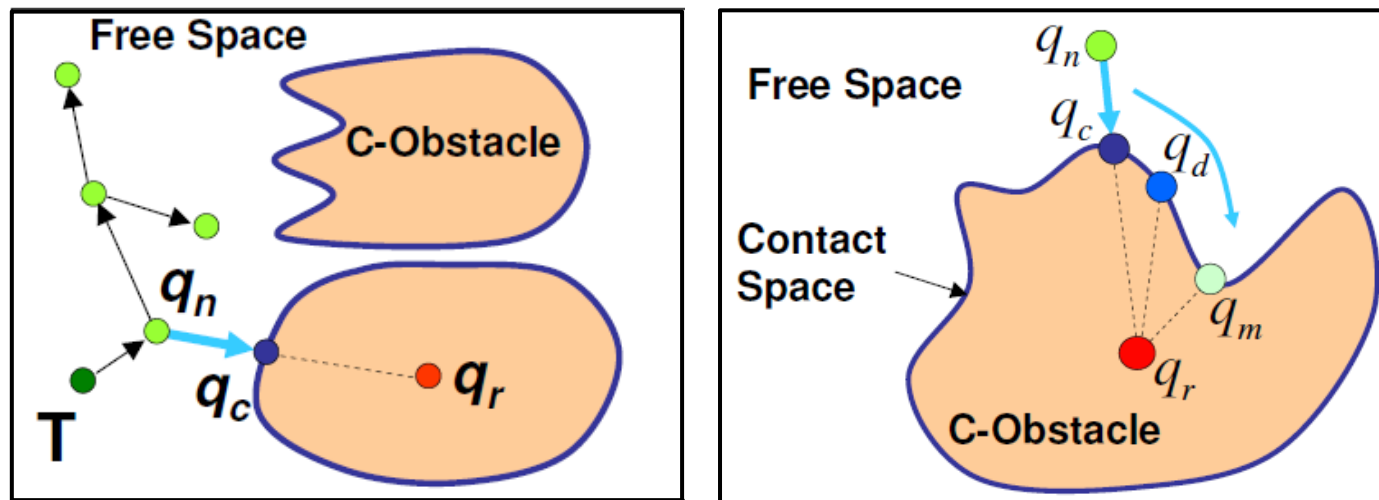
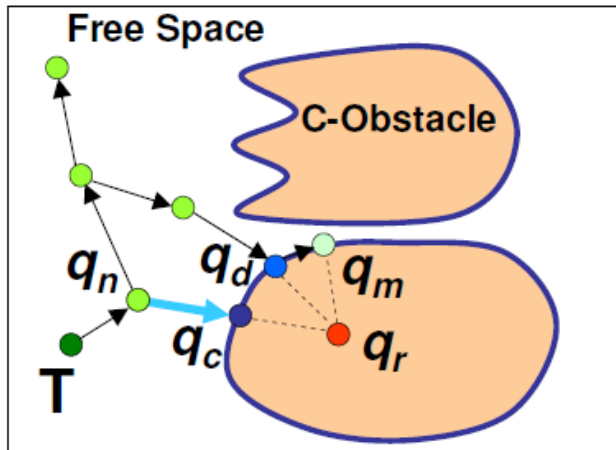


image from [Zhang & Manocha 08]

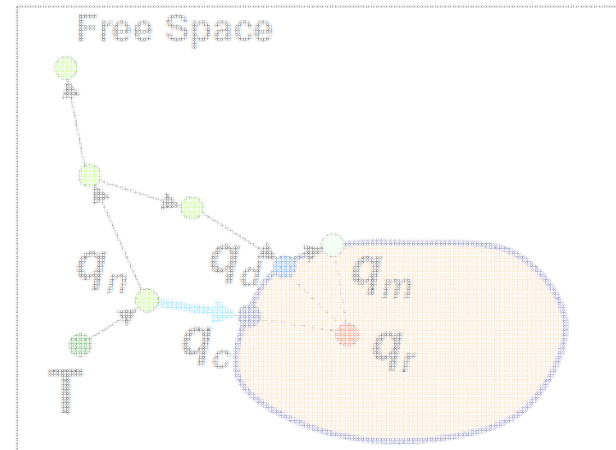
- General characteristic:**  
Generates more samples near the boundary of obstacles



# RRRT: Pros and Cons

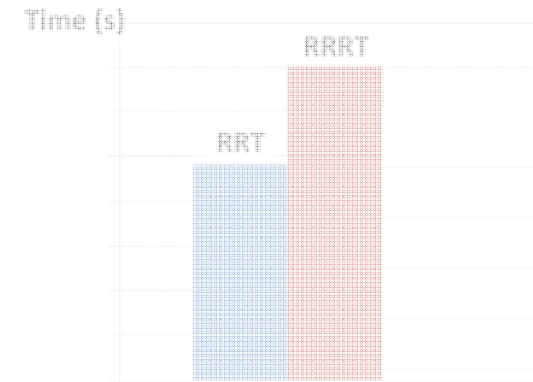
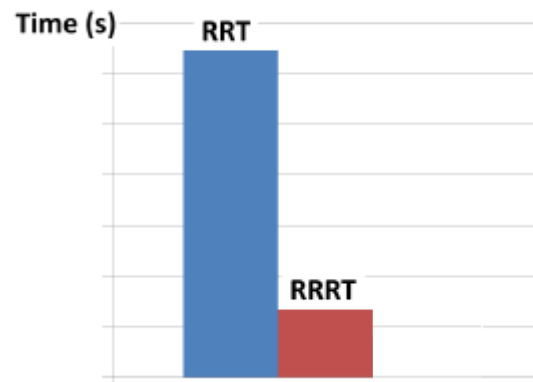


with narrow passages

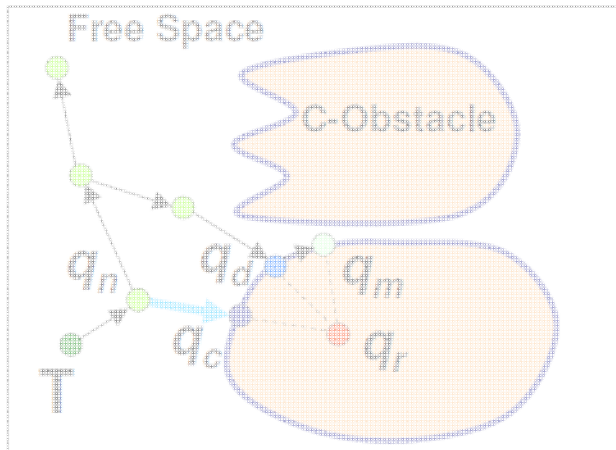


without narrow passages

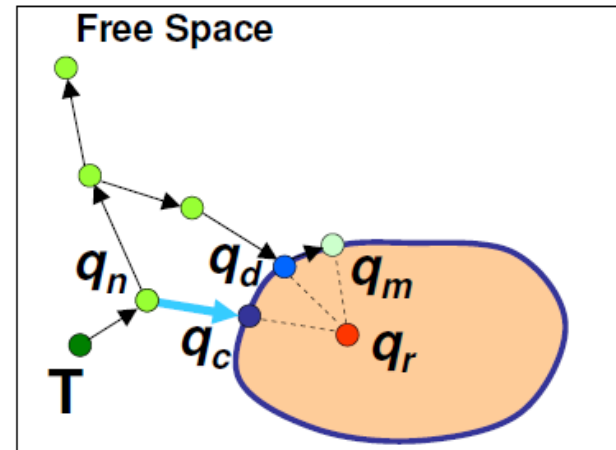
images from [Zhang & Manocha 08]



# RRRT: Pros and Cons

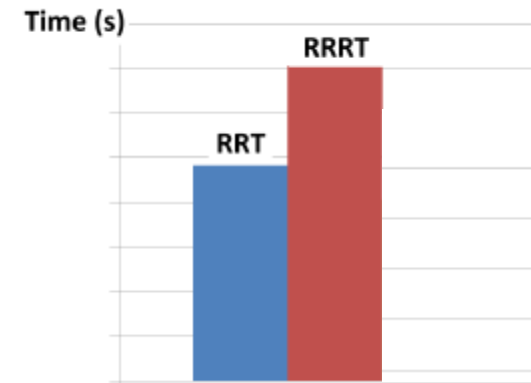
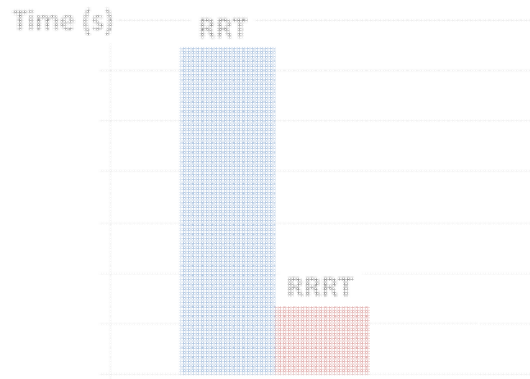


with narrow passages



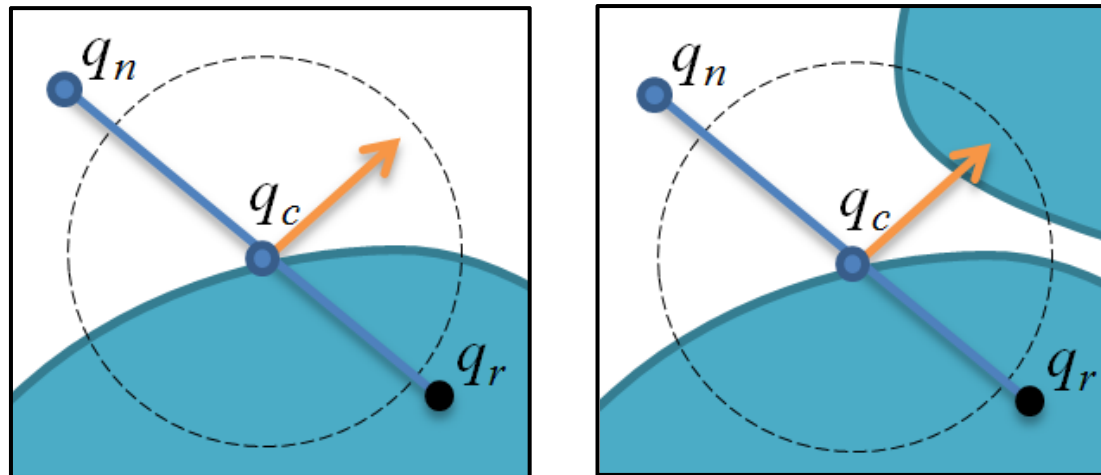
without narrow passages

images from [Zhang & Manocha 08]

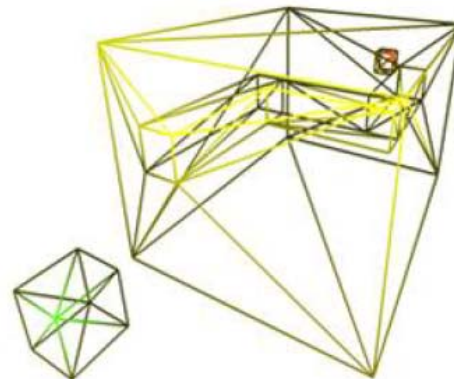
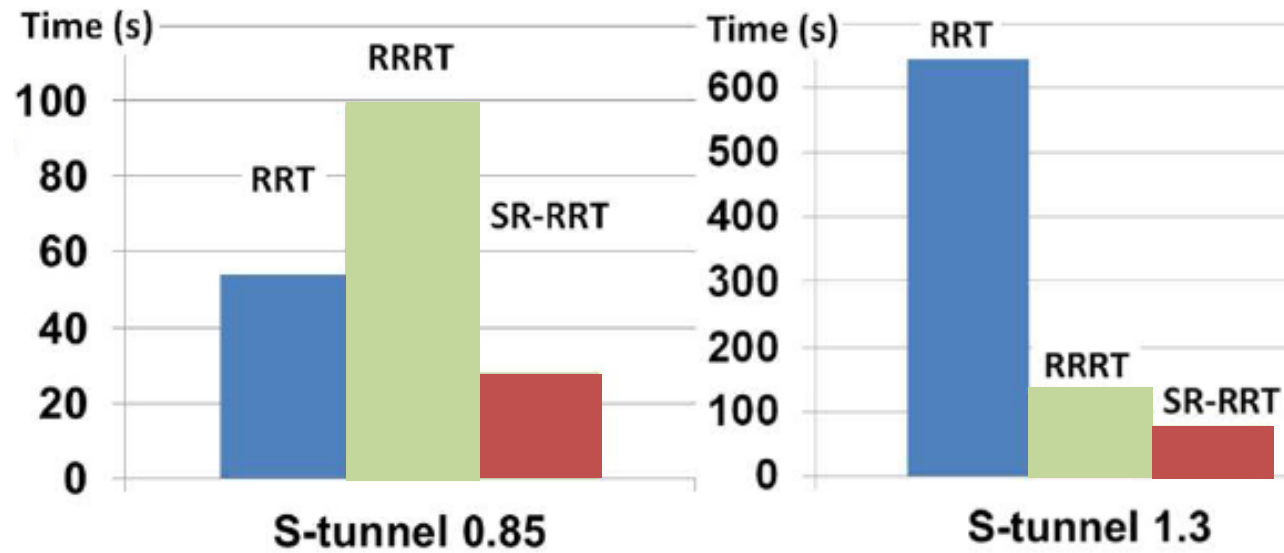


# Bridge line-test [Lee et al., T-RO 14]

- To identify narrow passage regions
- Bridge line-test
  1. Generate a random line
  2. Check whether the line meets any obstacle



# Results



Video

# Recent Works of Our Group

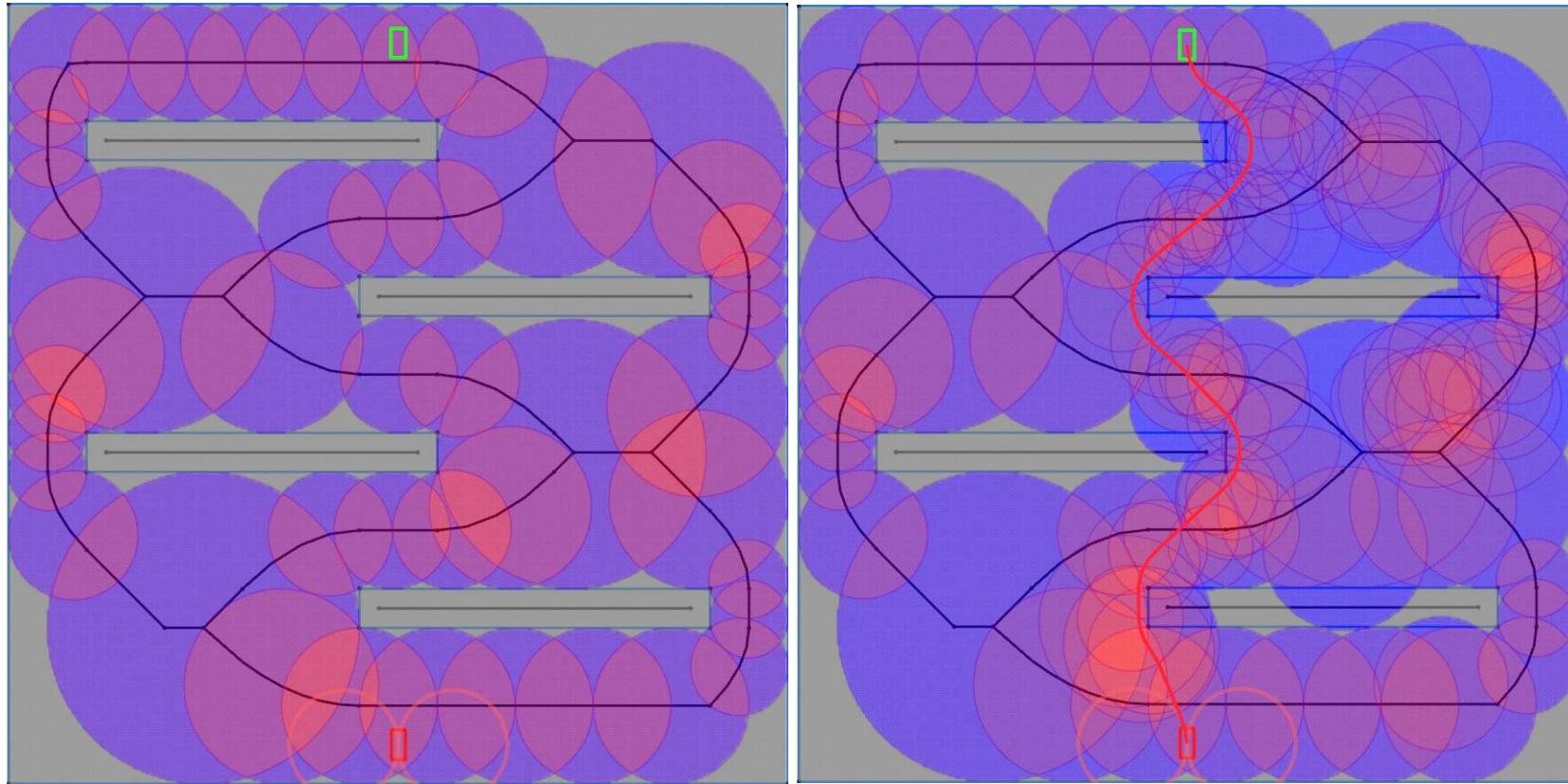
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- Handling narrow passages
- Improving low convergence to the optimal solution
  - Use the sampling cloud to indicate regions that lead to the optimal path
  - Cloud RRT\* : Sampling Cloud based RRT\* , Kim et al., ICRA 14
  - <http://sglab.kaist.ac.kr/CloudRRT/>

# Examples of Sampling Cloud

[Kim et al., ICRA 14]

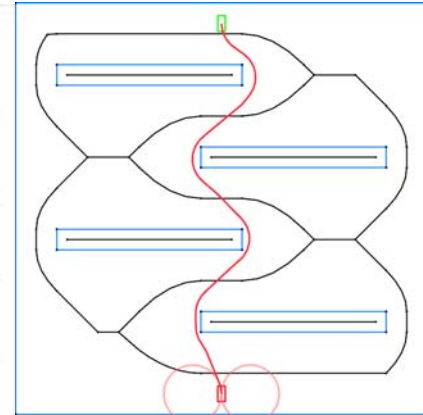
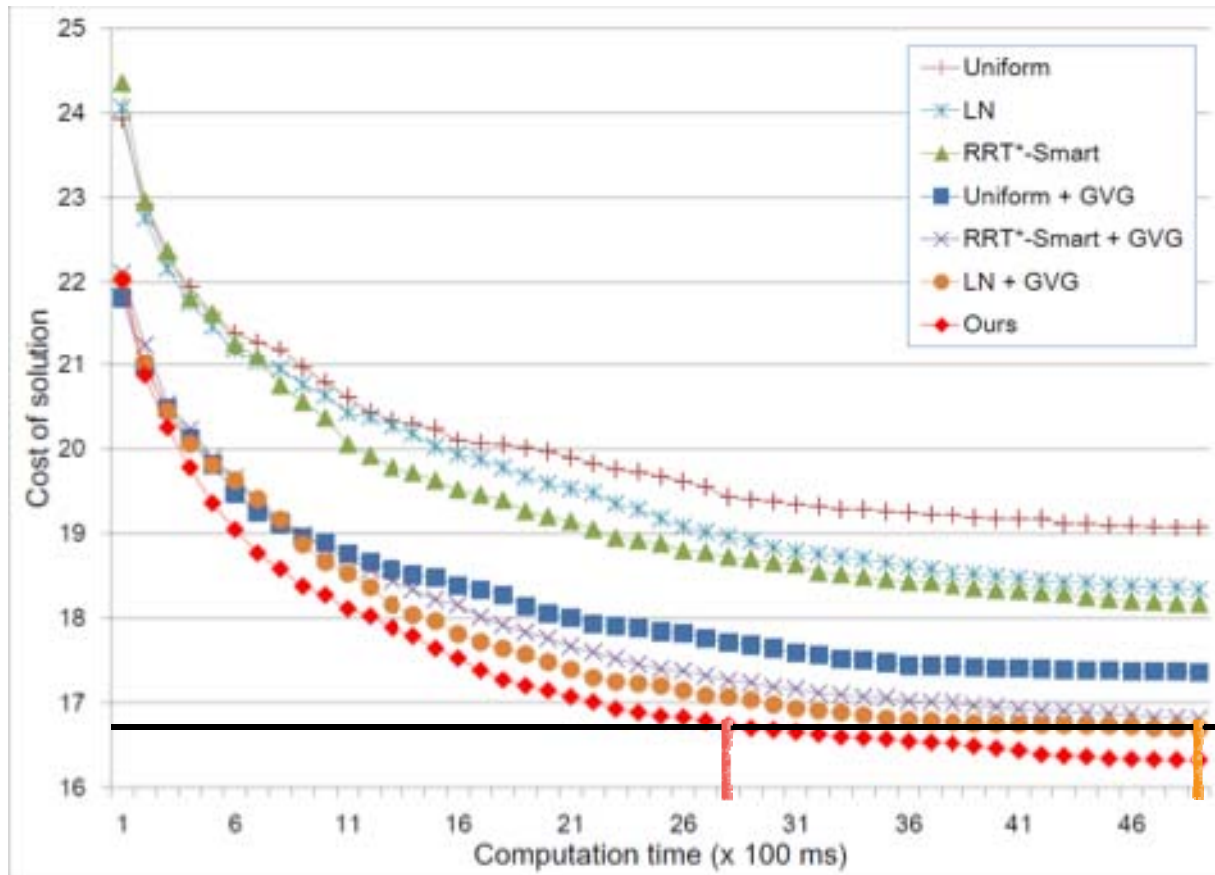


Initial state of sampling cloud

After updated several times

Video

# Results: 4 squares



**1.8X  
improvement**

# Recent Works of Our Group

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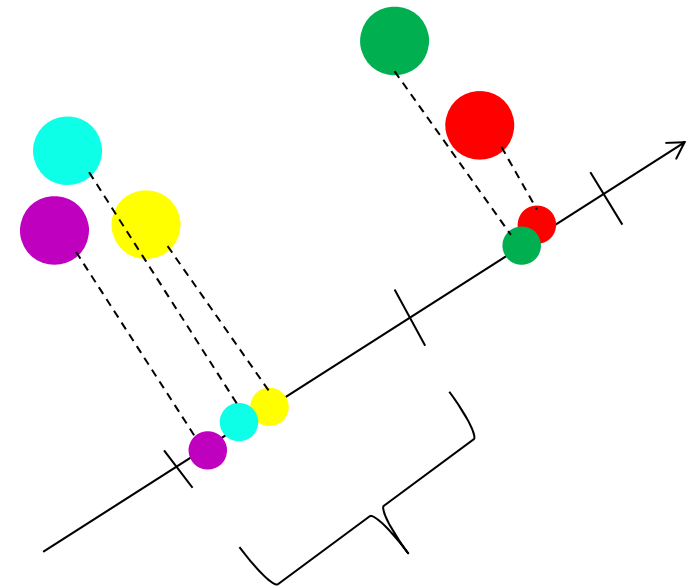
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- Handling narrow passages
- Improving low convergence to the optimal solution
- Accelerating nearest neighbor search
  - VLSH: Voronoi-based Locality Sensitive Hashing, Loi et al., IROS 13



# Background on Locality Sensitive Hashing (LSH)

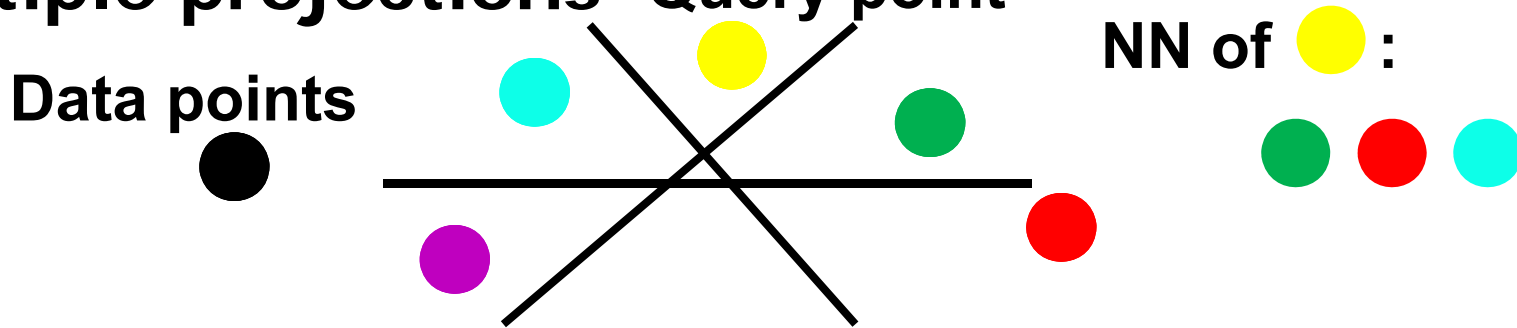
- Randomly generate a projection vector
- Project points onto vector
- Bin the projected points to a segment, whose width is  $w$ , i.e. quantization factor
- All the data in a bin has the same hash code



Quantization factor  $w$

# Background on LSH

- Multiple projections



$g_1$

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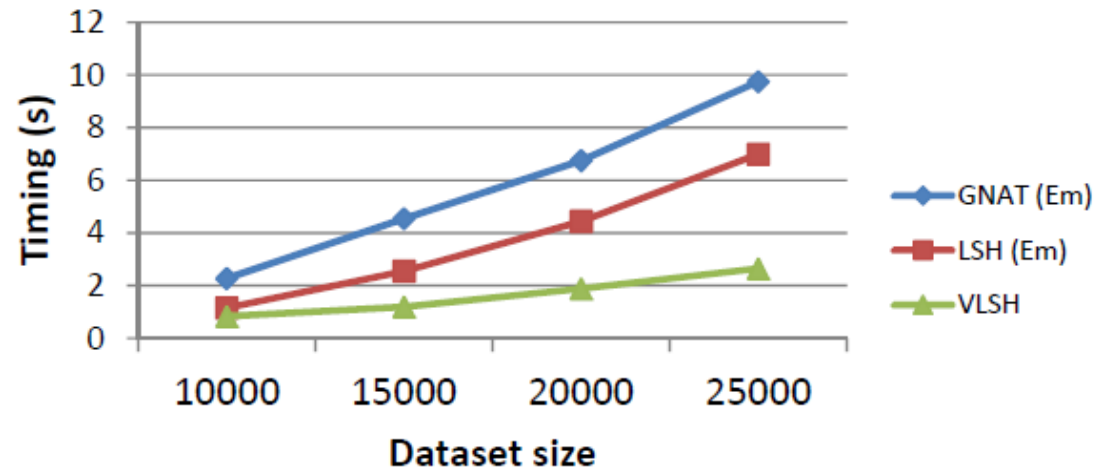
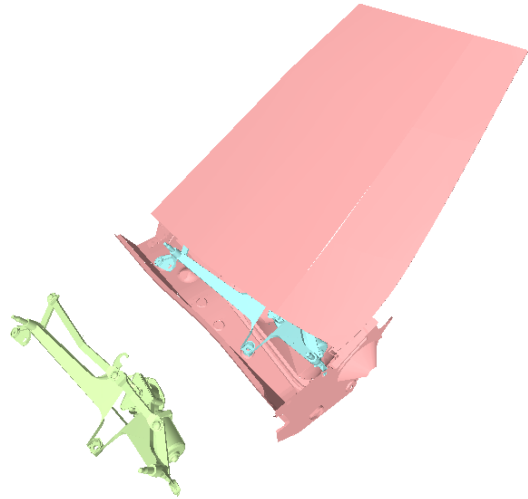
$g_2$

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$g_3$

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# Wiper: Performance Evaluation



- **VLSH vs. GNAT (Em):**
  - 3.7x faster
- **VLSH vs. LSH (Em):**
  - 2.6x faster

# Class Objectives were:

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- Understand the RRT technique and its recent advancements
  - RRT\* for optimal path planning
  - Kinodynamic planning

# No More HWs on:

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- Paper summary and questions submissions
- Instead:
  - Focus on your paper presentation and project progress!

# Summary

