CS686:
Classic Motion Planning Methods

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(윤성의)

Course URL:
http://sgvr.kaist.ac.kr/~sungeui/MPA

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Class Objectives

- **Classic motion planning approaches**
  - **Roadmap**
  - **Cell decomposition**
  - **Potential field**

Classic Path Planning Approaches

- **Roadmap**
  - **Represent the connectivity of the free space by a network of 1-D curves**
- **Cell decomposition**
  - **Decompose the free space into simple cells and represent the connectivity of the free space by the adjacency graph of these cells**
- **Potential field**
  - **Define a function over the free space that has a global minimum at the goal configuration and follow its steepest descent**

Classic Path Planning Approaches

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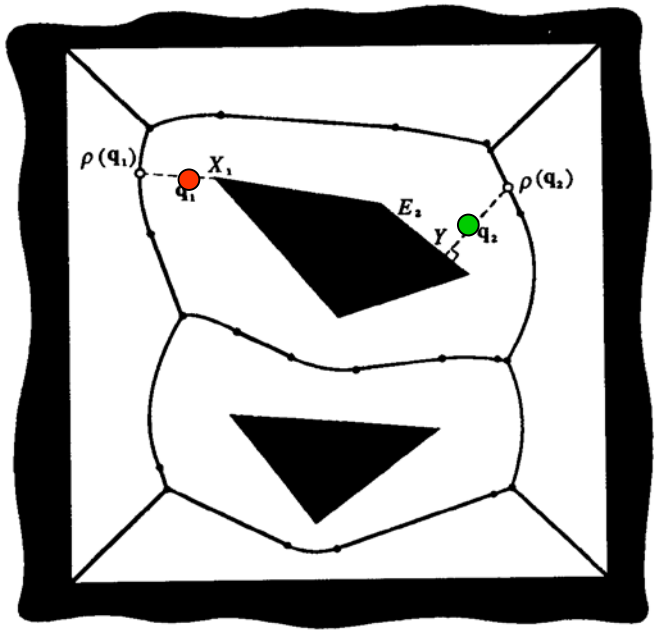
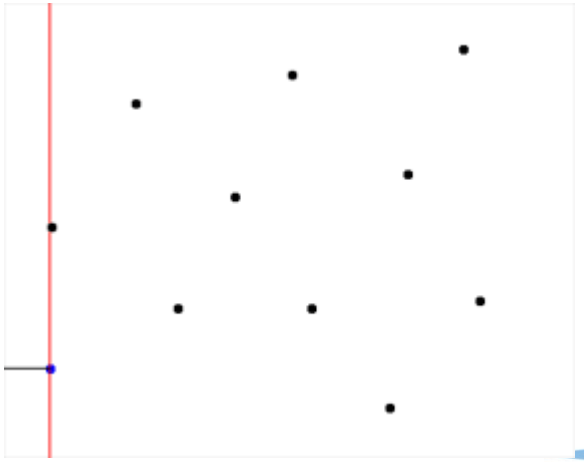
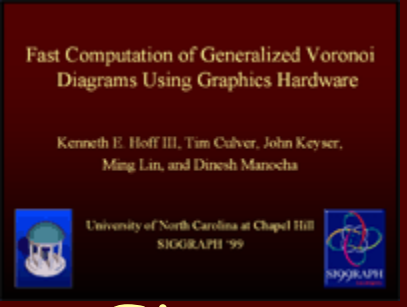
Roadmap Methods

- Visibility Graph
  - Shakey project, SRI [Nilsson 69]
- Voronoi diagram
  - Introduced by computational geometry researchers
  - Generate paths that maximize clearance
  - O(n log n) time and O(n) space for 2D points

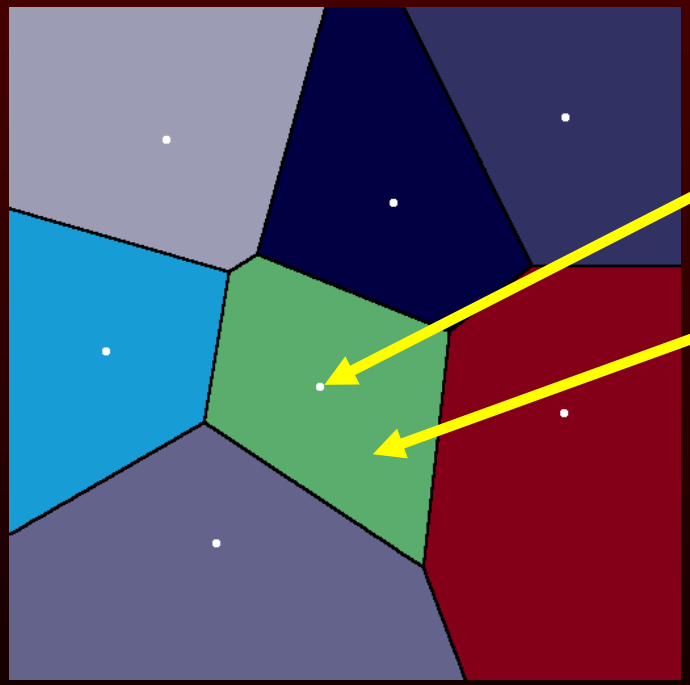


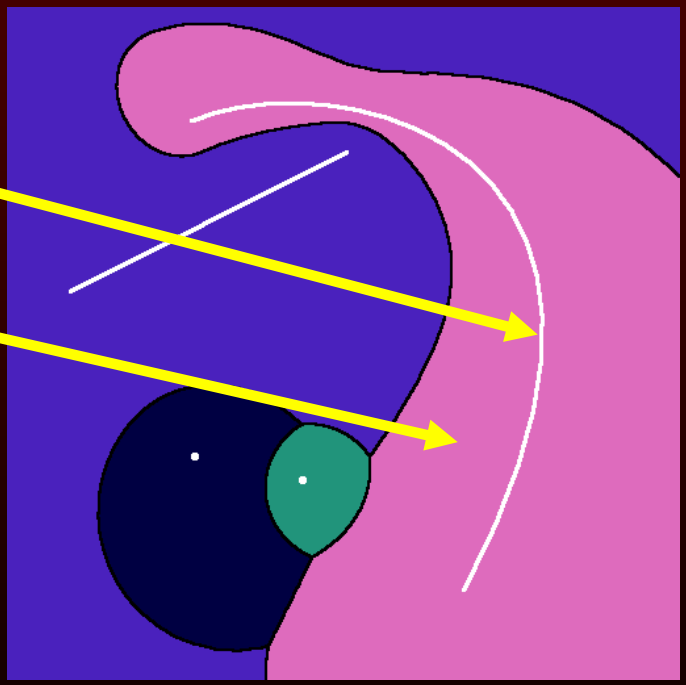




What is a Voronoi Diagram?

Given a collection of geometric primitives, it is a subdivision of space into cells such that all points in a cell are *closer* to one primitive than to any other

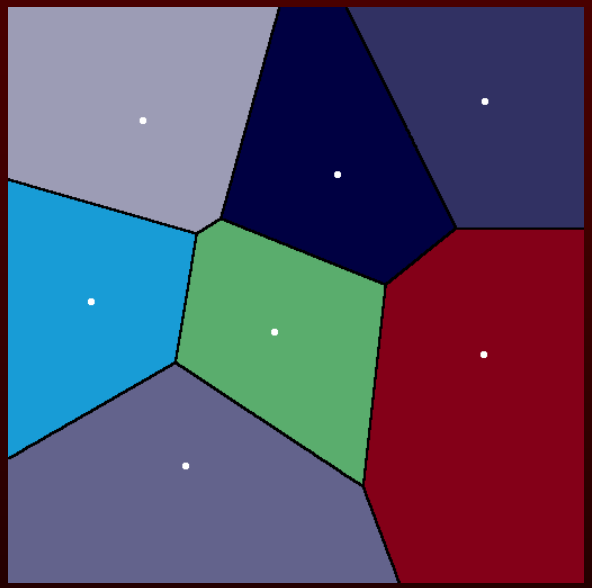




Adopted from:
Fast Computation of GVD using Graphics Hardware, Hoff et al., SIGGRAPH 1999

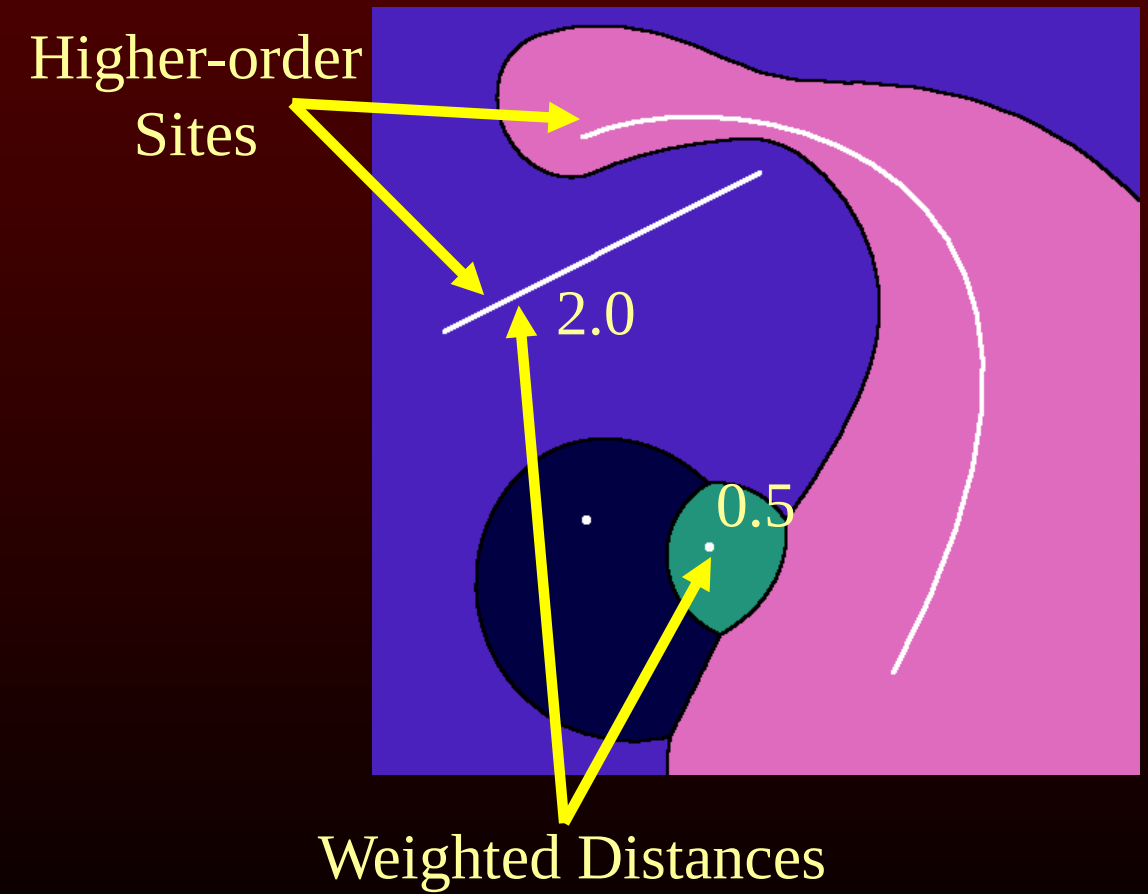
Ordinary

- Point sites
- Nearest Euclidean distance



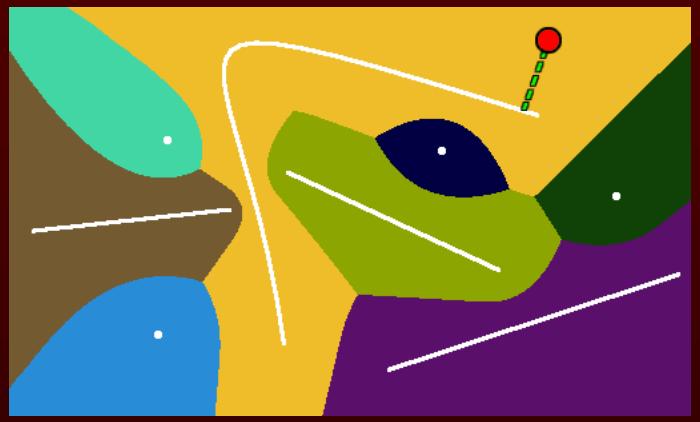
Generalized

- Higher-order site geometry
- Varying distance metrics

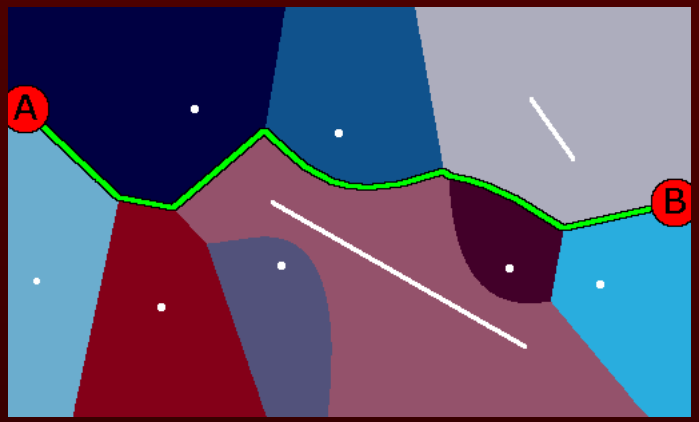


What Makes Them Useful?

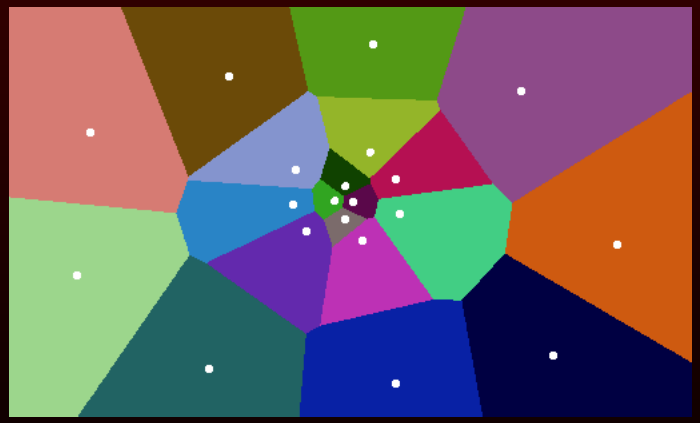
“Ultimate” Proximity Information



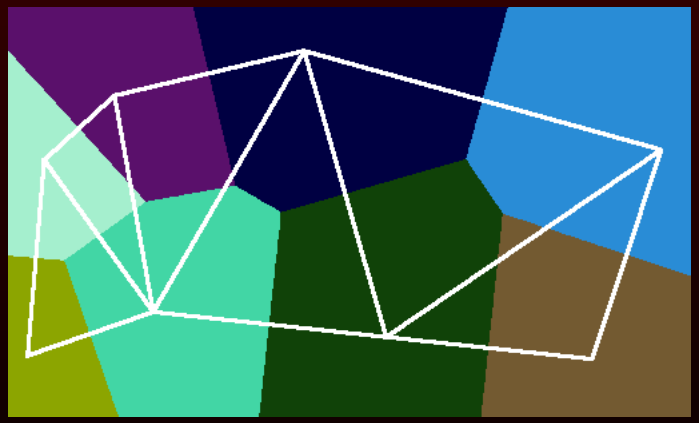
Nearest Site



Maximally Clear Path



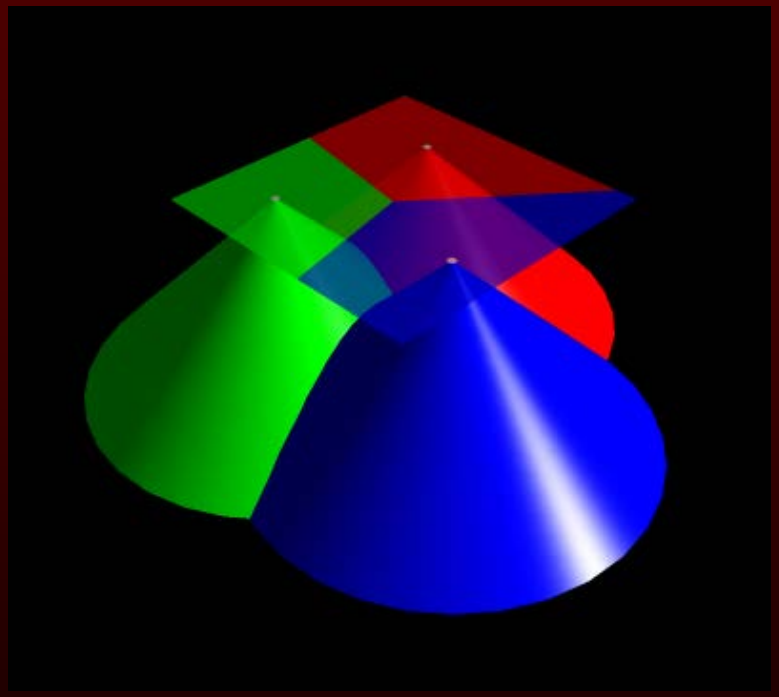
Density Estimation



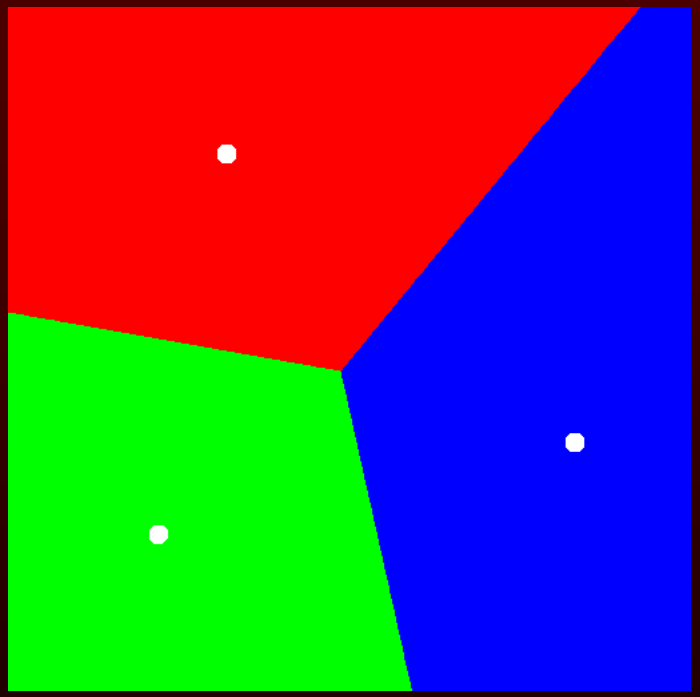
Nearest Neighbors

Cone Drawing

To visualize Voronoi diagram for points in 2D…



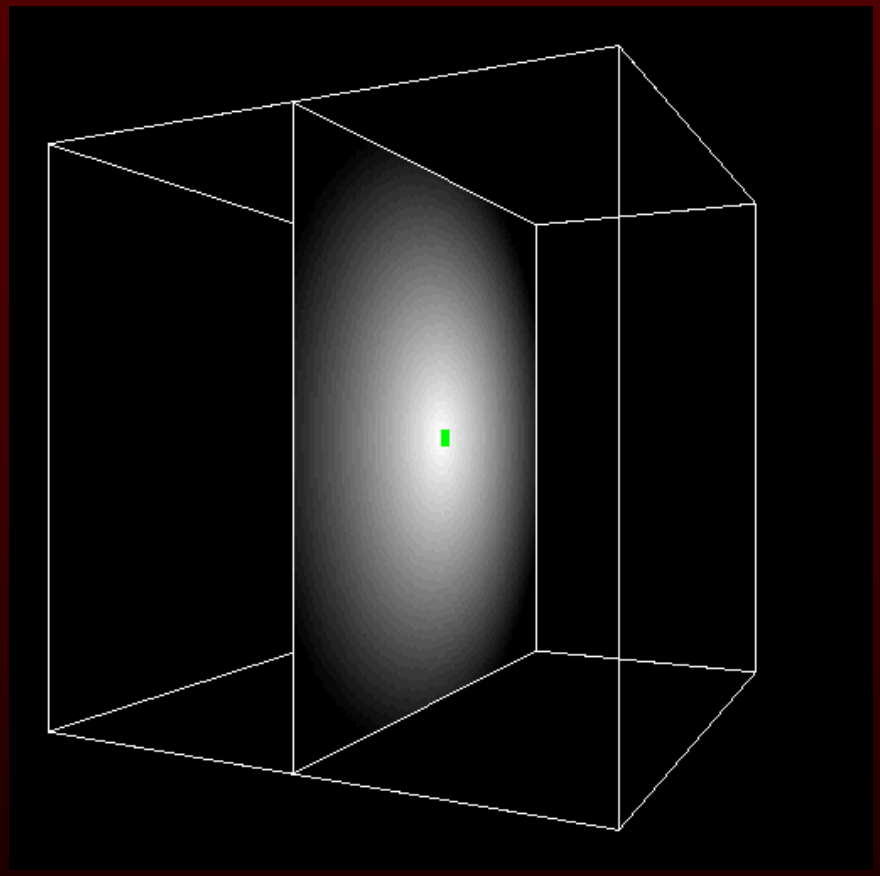
Perspective, 3/4 view



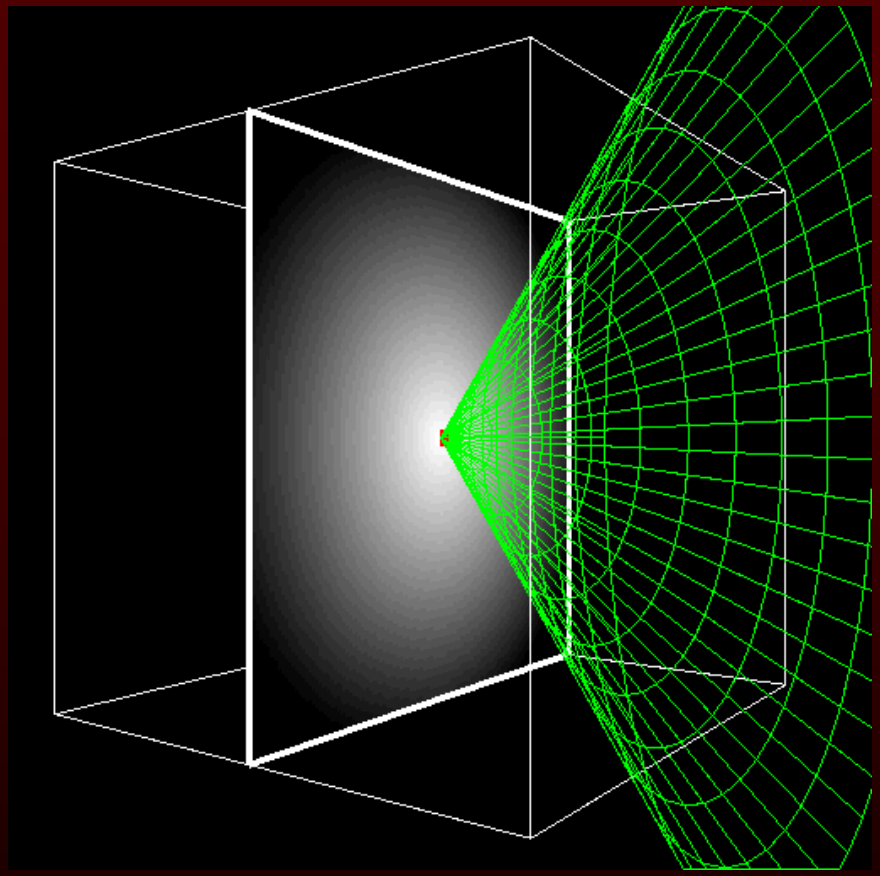
Parallel, top view

Dirichlet 1850 & Voronoi 1908

3D Voronoi Diagrams



Slices of the distance function for a 3D point site



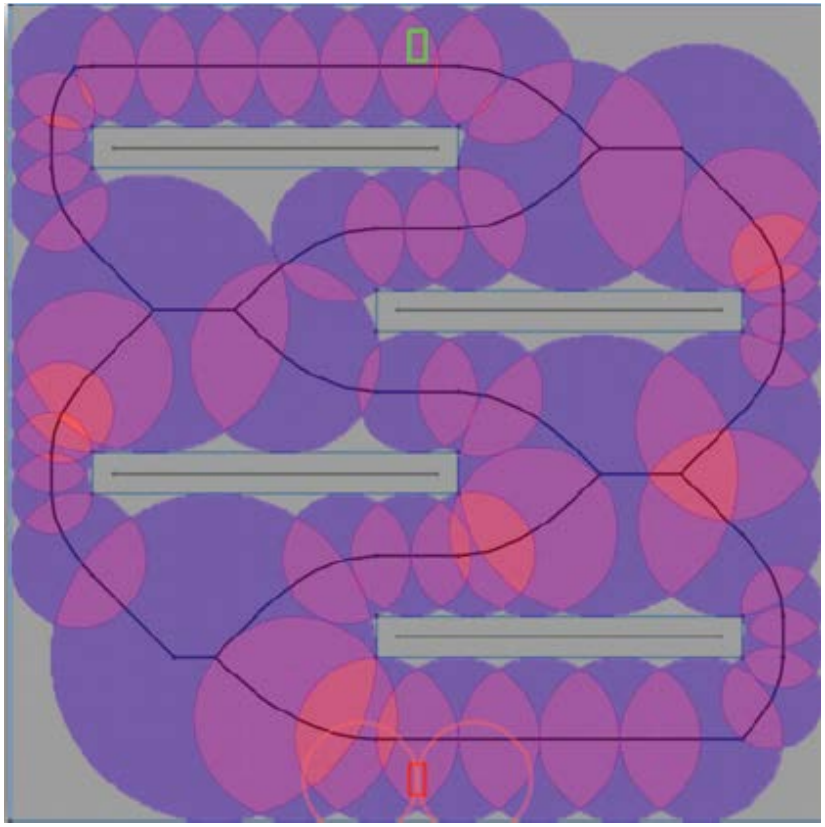
Distance meshes used to approximate slices

Other Roadmap Methods

- **Visibility graph**
- **Voronoi diagram**
- **Silhouette**
  - **First complete general method that applies to spaces of any dimension and is singly exponential in # of dimensions [Canny, 87]; e.g., $O\left(n^{2^{f(k)}}\right) \rightarrow O(n^k)$**
- **Probabilistic roadmaps**

Cloud RRT* [Kim et al., ICRA]

- **Use Voronoi diagram to bias sampling for achieving better convergence to optimal path**



https://www.youtube.com/watch?v=NSuDtd1amC4

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Classic Path Planning Approaches

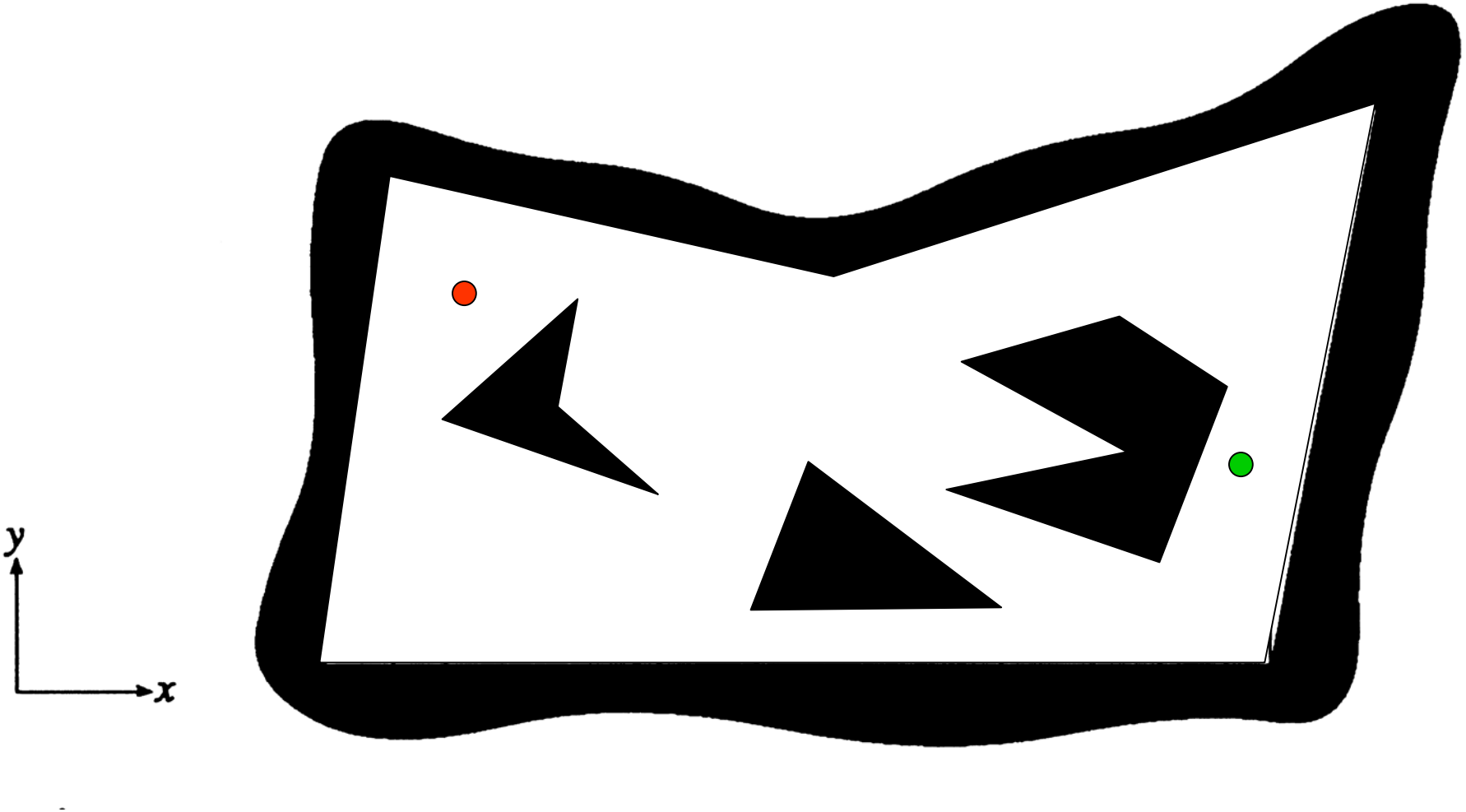
- **Roadmap**
  - **Represent the connectivity of the free space by a network of 1-D curves**
- **Cell decomposition**
  - **Decompose the free space into simple cells and represent the connectivity of the free space by the adjacency graph of these cells**
- **Potential field**
  - **Define a function over the free space that has a global minimum at the goal configuration and follow its steepest descent**

Cell-Decomposition Methods

- **Two classes of methods:**
  - **Exact and approximate cell decompositions**

- **Exact cell decomposition**
  - **The free space F is represented by a collection of non-overlapping cells whose union is exactly F**
  - **Example: trapezoidal decomposition**

Trapezoidal Decomposition



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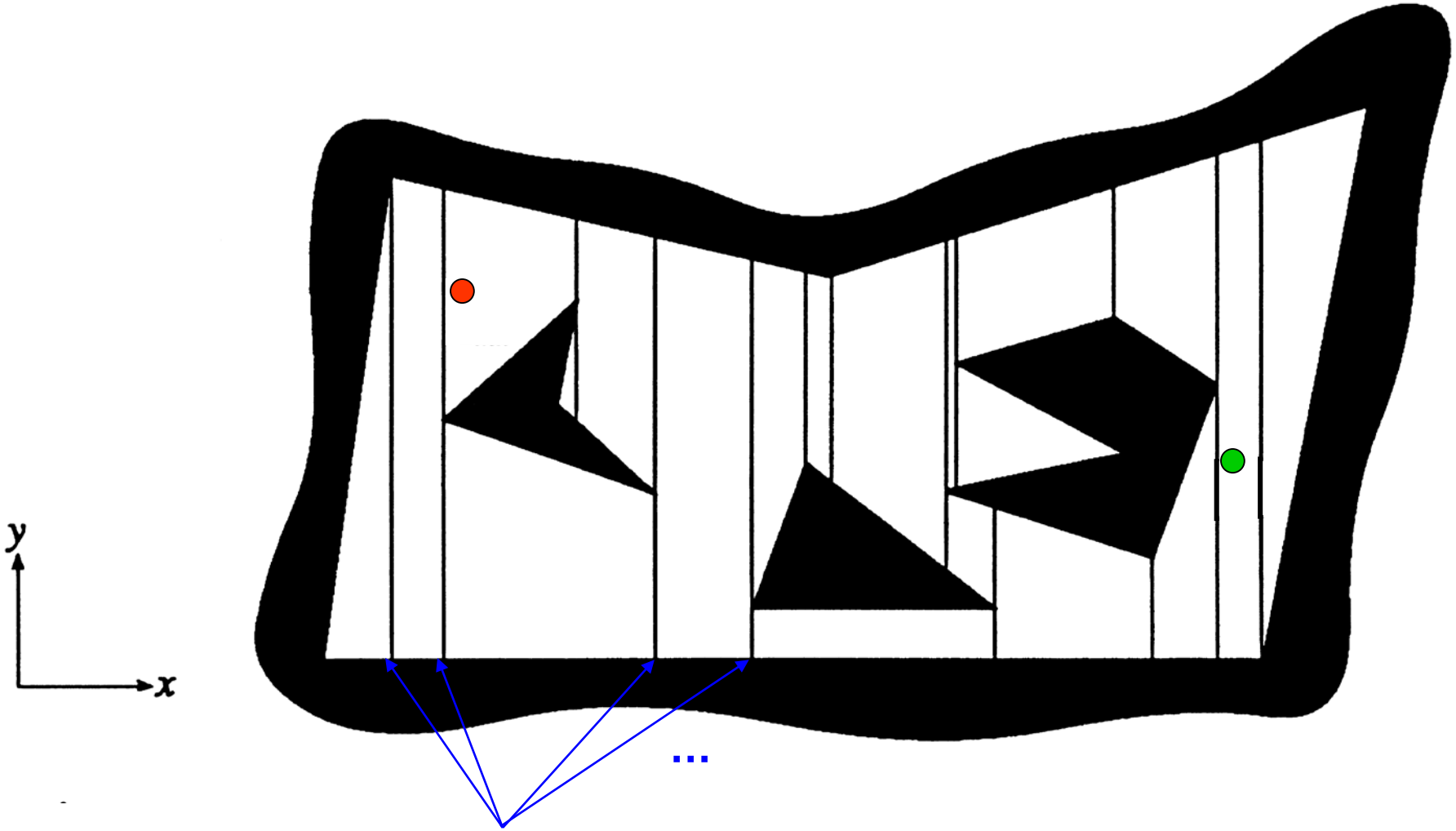
Trapezoidal Decomposition



critical events → criticality-based decomposition

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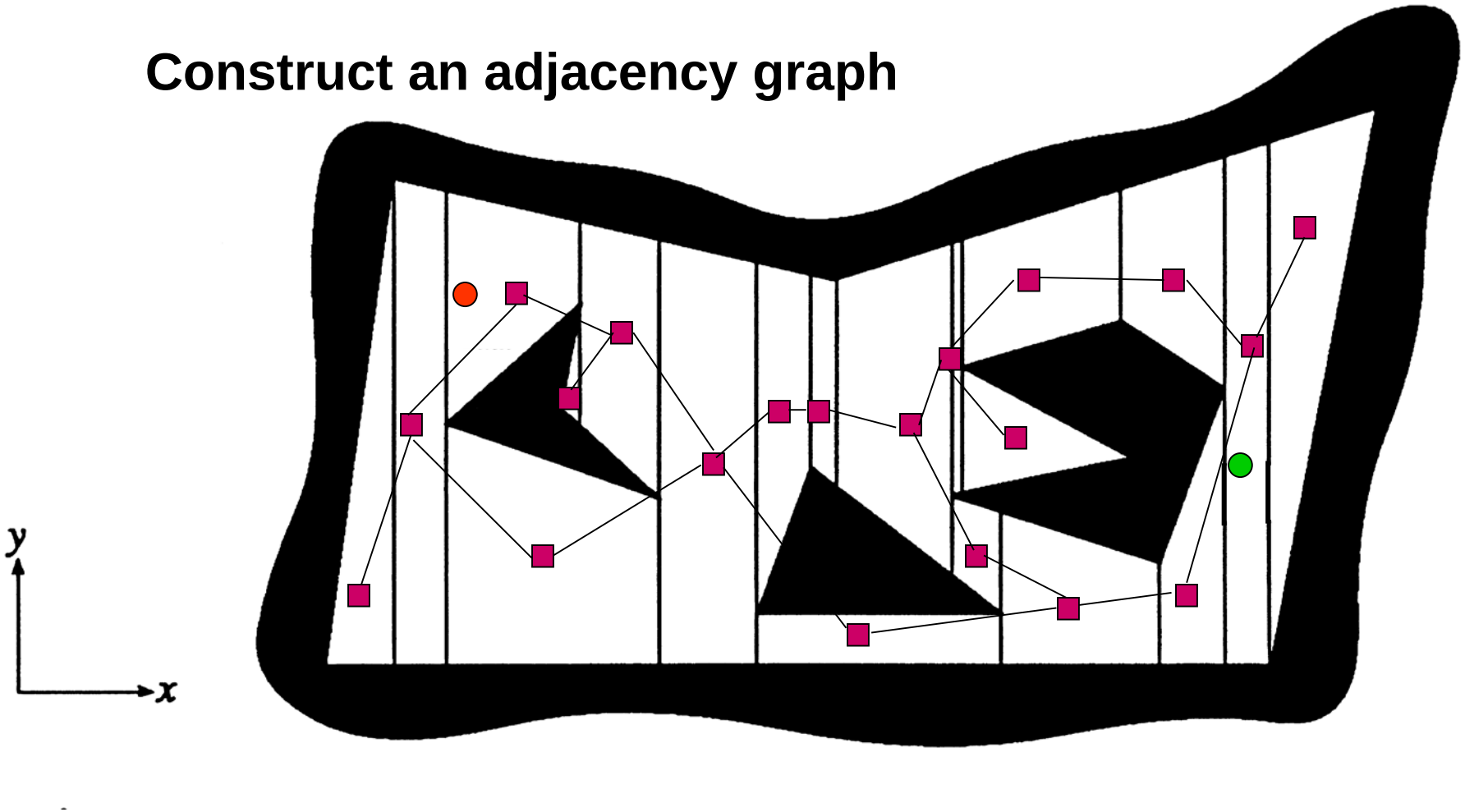
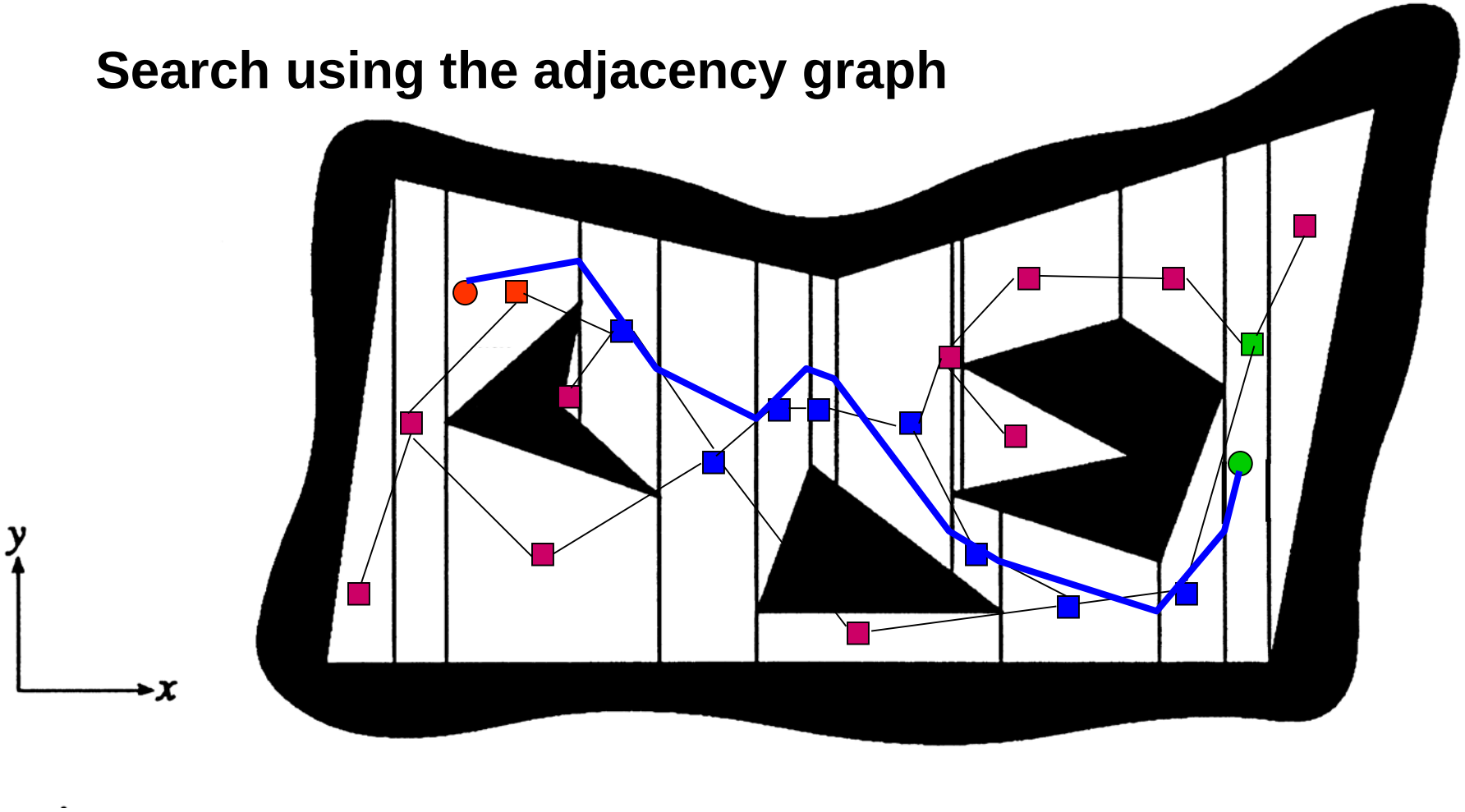
Trapezoidal Decomposition

Construct an adjacency graph



Trapezoidal Decomposition

Search using the adjacency graph



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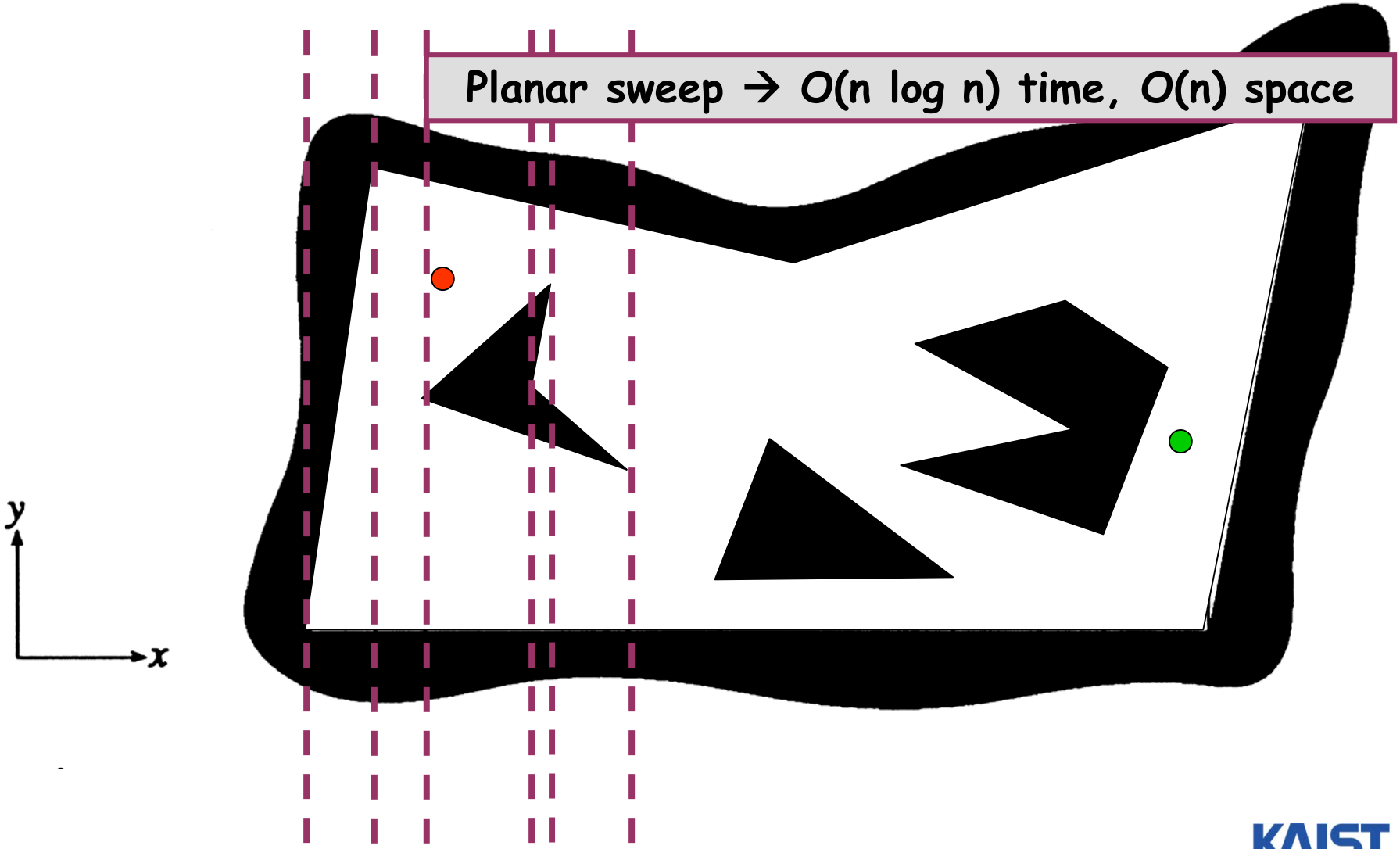
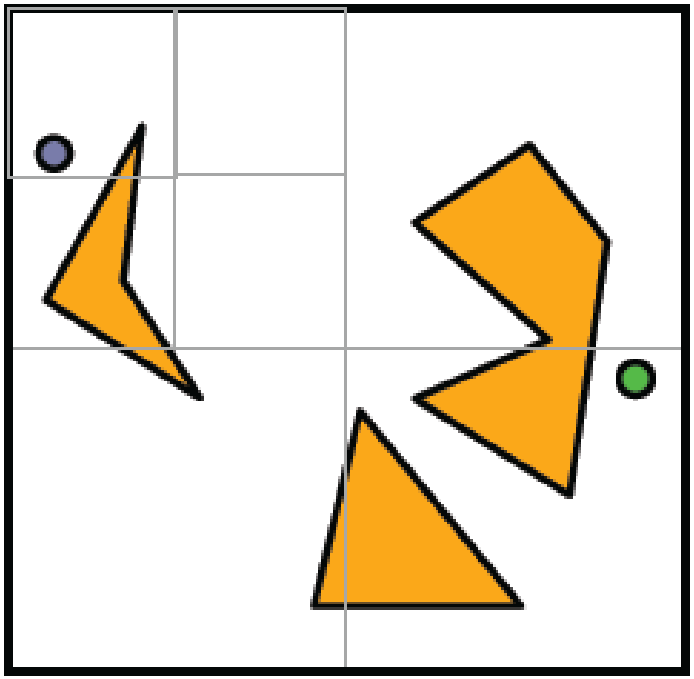
Trapezoidal Decomposition

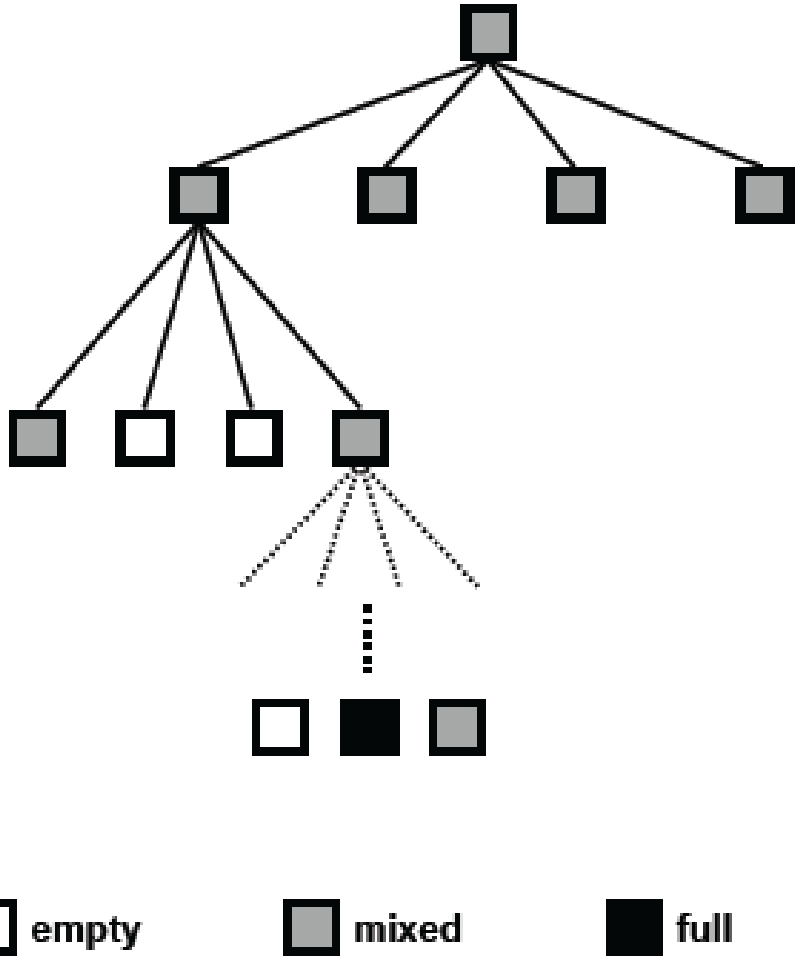


Cell-Decomposition Methods

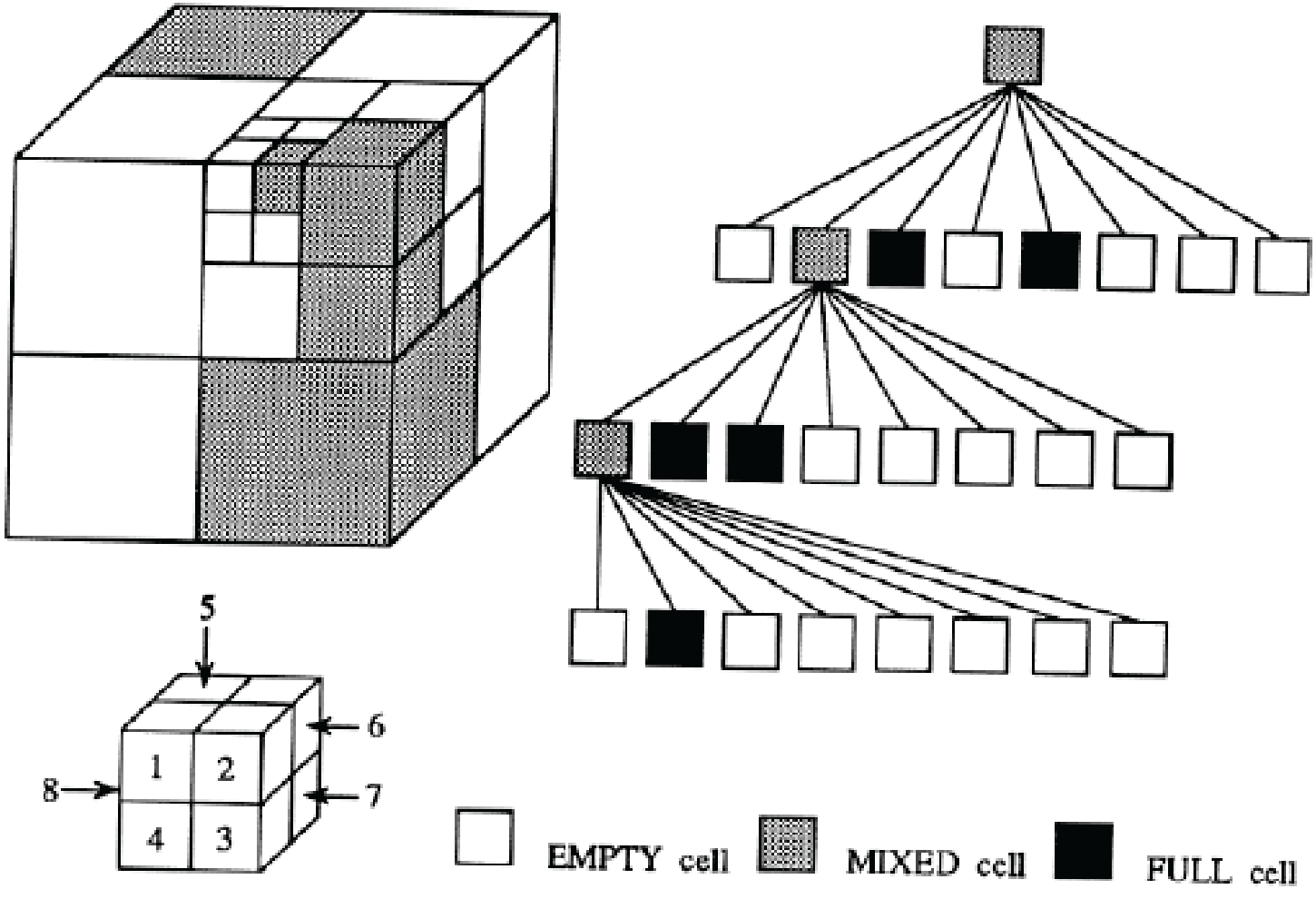
- **Two classes of methods:**
  - **Exact and approximate cell decompositions**
- **Exact cell decomposition**
- **Approximate cell decomposition**
  - **The free space F is represented by a collection of non-overlapping cells whose union is contained in F**
  - **Cells usually have simple, regular shapes (e.g., rectangles and squares)**
  - **Facilitates hierarchical space decomposition**

Quadtree decomposition



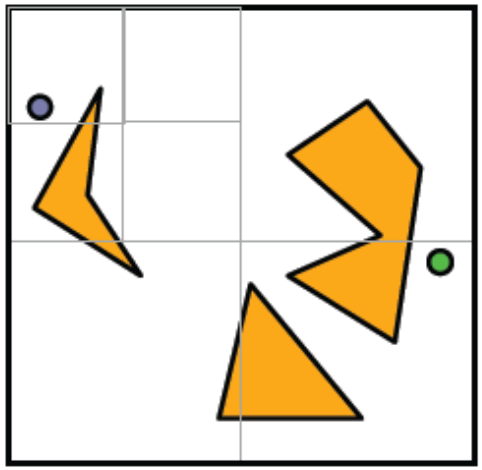


Octree decomposition



Sketch of Algorithm

1. Decompose the free space F into cells
2. Search for a sequence of **mixed** or **free** cells that connect that initial and goal positions
3. Further decompose the mixed
4. Repeat 2 and 3 until a sequence of **free** cells is found



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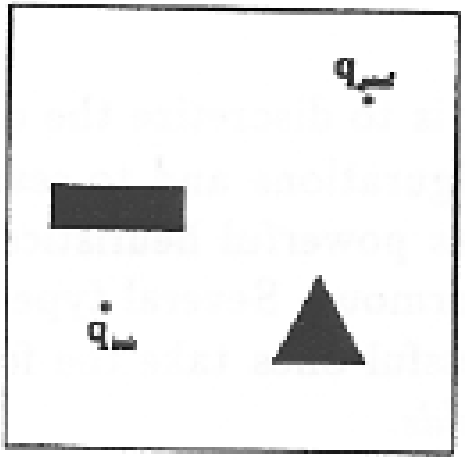
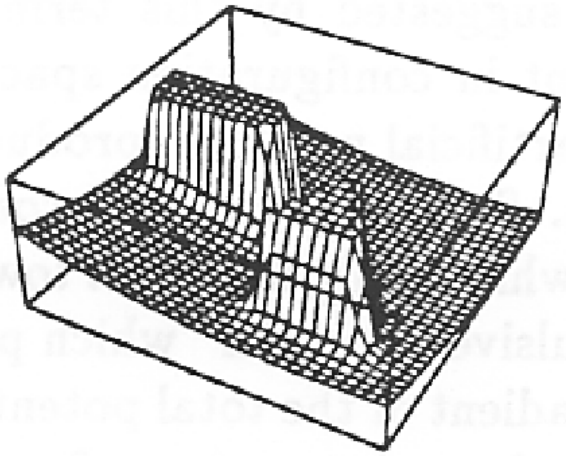
Potential Field Methods

- **Initially proposed for real-time collision avoidance [Khatib, 86]**
  - **Use a scalar function, potential field, over the free space**
  - **Compute a force proportional to the negated gradient of the potential field**



Workspace



A potential field

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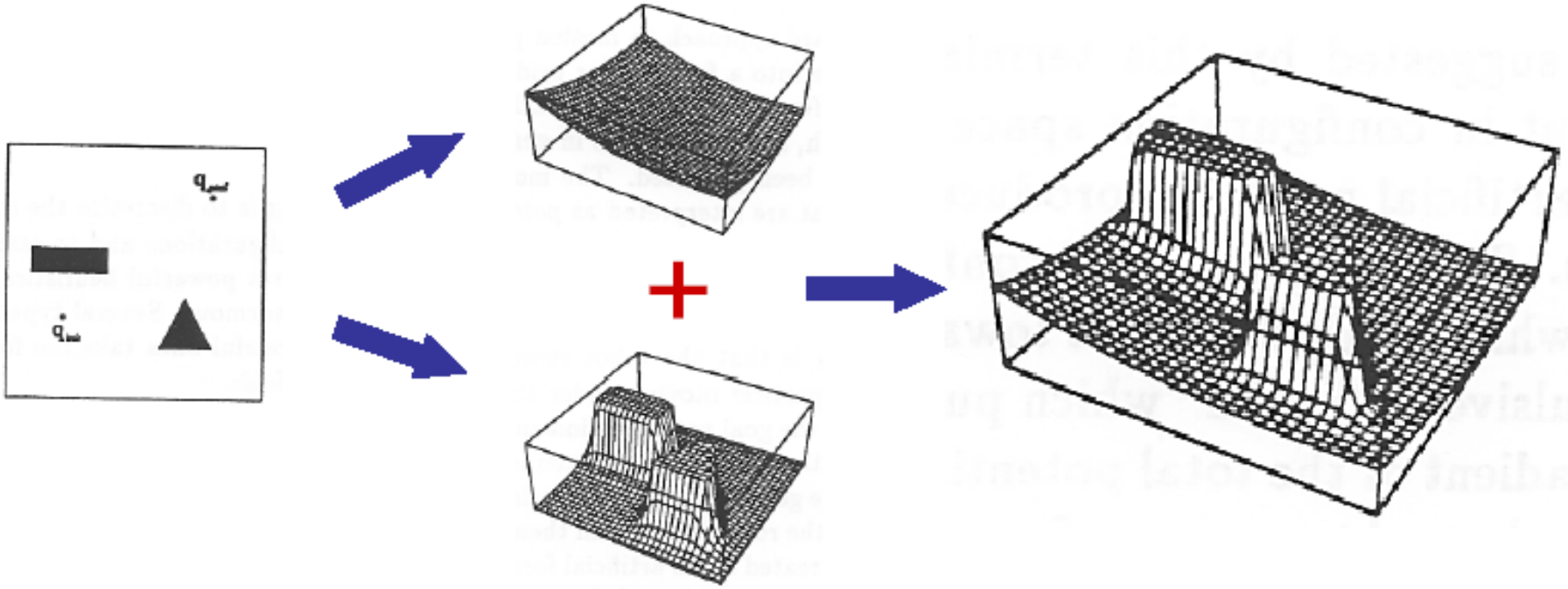
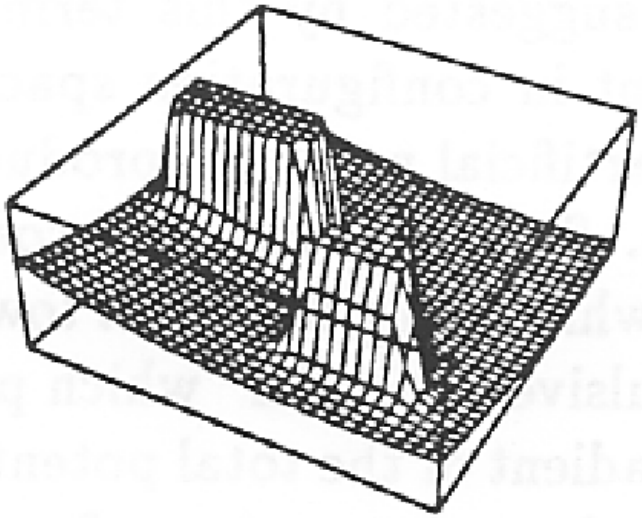
Attractive and Repulsive fields

Attractive field towards the goal



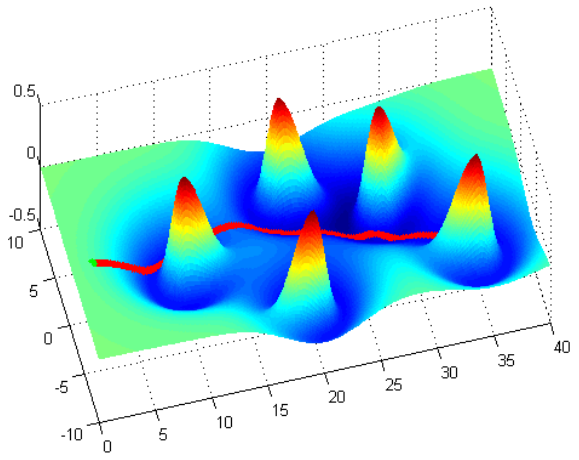
Repulsive field away from obstacles

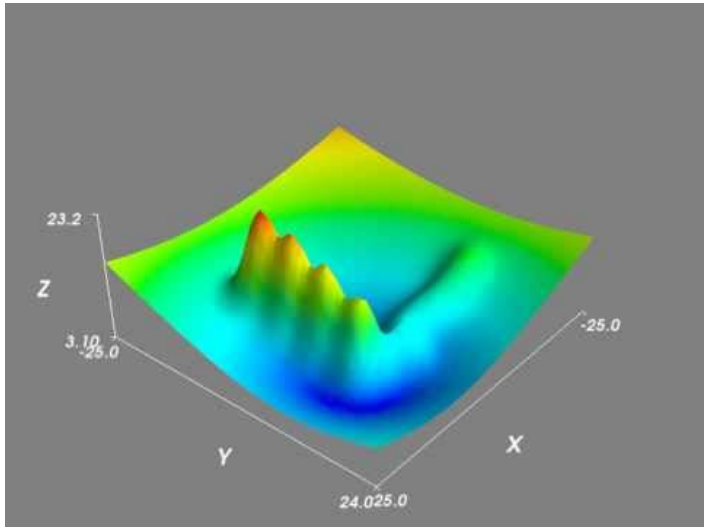
Ideal Potential Field



- **The ideal one**
  - **Has the global minimum at the goal**
  - **Has no local minima**
  - **Grows to infinity near obstacles**
  - **Is smooth**
- **Can we compute the one?**

Local Minima





Svenstrup

- **What can we do?**
  - **Escape from local minima by taking random walks**

Sketch of Algorithm

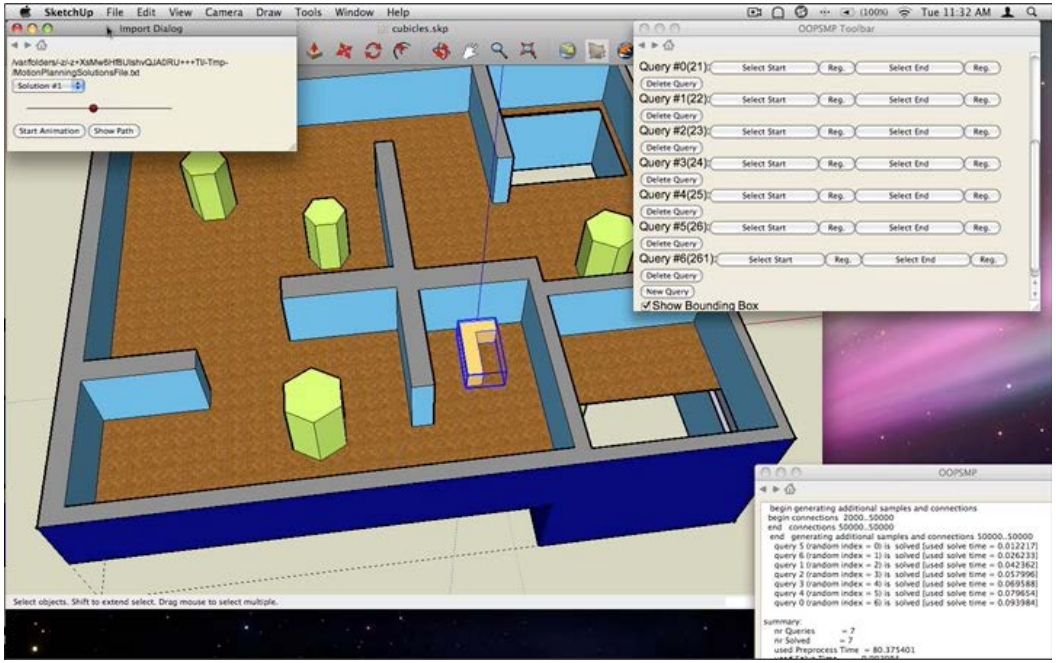
- Place a regular grid G over the configuration space
- Compute the potential field over G
- Search G using a best-first algorithm with potential field as the heuristic function

Completeness

- A complete motion planner always returns a solution when one exists and indicates that no such solution exists otherwise
  - Is the visibility algorithm complete? Yes
  - How about the exact cell decomposition algorithm and the potential field algorithm?

Homework: PA1

- Install Open Motion Planning Library (OMPL)
- Create a scene and a robot
- Find a collision-free path and visualize the path



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Homework

- **Deadline: 11:59pm, Mar.-26**
- **Submit at KLMS:**
  - **An image that shows a scene with a robot with a computed path**

Conf. Deadline

- **ICRA: Sep.-15 2019**
- **RSS: Jan(?), IROS: March**

Class Objectives were:

- **Classic motion planning approaches**
  - **Roadmap**
  - **Cell decomposition**
  - **Potential field**

Homework

- **Browse 2 ICRA / IROS / RSS / CoRL / TRO / IJRR papers**
  - **Prepare two summaries and submit at the beginning of every Tue. class, or**
  - **Submit it online before the Tue. Class**

- **Example of a summary (just a paragraph)**

  Title: XXX XXXX XXXX
  Conf./Journal Name: ICRA, 2015
  Summary: this paper is about accelerating the performance of collision detection. To achieve its goal, they design a new technique for reordering nodes, since by doing so, they can improve the coherence and thus improve the overall performance.

Homework for Every Class

- **Go over the next lecture slides**
- **Come up with one question on what we have discussed today and submit at the end of the class**
  - **1 for typical questions**
  - **2 for questions with thoughts or that surprised me**
- **Write a question 3 times before the mid-term exam**

Next Time….

- **Configuration spaces**