Informative Path Planning for Source Localization

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I. Introduction

SOURCE localization is an important problem that has been studied in various contexts. For example, in the underwater domain, acoustic source localization has applications in search & rescue (e.g. finding the black-box of a sunken aircraft), subsea operations, and in defense & coastal security. While interaural phase difference (IPD) and interaural time difference (ITD) techniques are well-studied and effective, they require multiple spatially separated acoustic sensors with accurate time synchronization. The need for time synchronization can be relaxed through the use of interaural level difference (ILD) techniques [1], but the need for large spacing between sensors limits the use of such sensors on small robots. Moreover, if source localization could be performed using a single low fidelity acoustic sensor, the cost and complexity of robots for source localization applications could be greatly reduced.

In the absence of the multipath propagation, signal intensity generally decreases monotonically with propagation range. A signal source could be correctly localized by using gradient ascent method to find the maximum intensity. However, signals in many practical conditions experience multipath propagation. The recevied signal intensity fluctuates significantly because of constructive and destructive interference from various propagation paths. Under such condition, the intensity pattern has several local maxima and gradient ascent performs poorly. If the multipath propagation of the signal from the source can be accurately modeled, a few measurements at different locations may be sufficient to determine the location of the source. This basic idea has been explored in the matched field processing (MFP) literature [2].

The key idea in being able to use a single sensor for source localization is to mount it on a robot, and to use the motion of the robot to spatially sample the field produced by the source, and then apply MFP techniques. The natural next question is how should the robot move in order to get the best localization in the shortest possible time? This important question is the focus of our paper. While the algorithm we propose is general and may be used with any kind of signal, we specifically focus on acoustic signals in this paper for concreteness.

The topic of path planning for source localization has not been treated extensively in literature, but some researchers have explored related ideas. In [3], an algorithm for radio frequency source localization based on the received signal intensity by multiple unmanned aerial vehicles (UAV) is proposed for a non-line-of-sight propagation conditions. The authors of [4] propose a method to localize an acoustic source using emergent behavior of a small team of robots, but the method performs poorly for single robots. In [5], the authors propose a Monte Carlo tree search based path-planning algorithm for a mobile robot to use a microphone array for sound source localization.

II. Path-planning Algorithm

We assume a robot with an acoustic intensity sensor operating in an environment where the acoustic propagation can be accurately modeled. The robot uses prior knowledge (e.g. source is within a specific area of interest, source is on the seabed, etc) to estimate an initial probability distribution for the source location. The robot starts at a known location and measures the acoustic intensity. It compares the measured intensity value against modeled values for potential source locations, and updates the probability distribution. The robot then moves and makes a measurement at another nearby location and repeats the process. We want to help the robot plan its move in such a way as to make the probability distribution as compact as possible, i.e., eliminate ambiguities in our knowledge of source location, in a short period of time.

At time step i, the probability of the source at a location $\boldsymbol{x}$ based on a set of measurements and corresponding robot's positions can be written as:

$$f(\boldsymbol{x}|\{(\boldsymbol{w}_j, z_j)\}\forall j \in [1, i]),$$

where $\boldsymbol{w}_j$ is the robot's position when measurement z_j was made. We henceforth use set $\mathcal{Y}_i \equiv (\{(\boldsymbol{w}_j, z_j)\}\forall j \in [1, i])$ for a more compact notation. By making the i^{th} measurement at location $\boldsymbol{w}_i$, the probability distribution of source location can be updated using Baye's Theorem:

$$\begin{aligned} f(\boldsymbol{x}|\mathcal{Y}_i) &= f(\boldsymbol{x}|\mathcal{Y}_{i-1} \cup (\boldsymbol{w}_i, z_i)) \\ &= \frac{f((\boldsymbol{w}_i, z_i)|\mathcal{Y}_{i-1}, \boldsymbol{x}) f(\boldsymbol{x}|\mathcal{Y}_{i-1})}{f((\boldsymbol{w}_i, z_i)|\mathcal{Y}_{i-1})} \\ &= \frac{f((\boldsymbol{w}_i, z_i)|\mathcal{Y}_{i-1}, \boldsymbol{x}) f(\boldsymbol{x}|\mathcal{Y}_{i-1})}{\sum_{\boldsymbol{x}} f((\boldsymbol{w}_i, z_i)|\mathcal{Y}_{i-1}, \boldsymbol{x}) f(\boldsymbol{x}|\mathcal{Y}_{i-1})}. \end{aligned} \tag{1}$$

We want the probability distribution $f(\boldsymbol{x}|\mathcal{Y}_i)$ to be as compact as possible, such that the uncertainty is minimal. We therefore wish to minimize the entropy of the distribution at each time step i, by selecting the next location for the robot to move to, essentially yielding an adaptively planned path. Do note that this approach is *greedy* and does not generate a globally optimal path, but still yields good results as we show in the next section.

Minimizing the entropy of the posterior distribution of source location is equivalent to maximizing the information gain in the Bayesian update. We therefore plan our path by choosing an action (direction for the robot to move) that leads to maximal information gain at each step. Before a robot makes a measurement at a given location, there is some prior uncertainly about the sound intensity at that location. Once the measurement is made, the uncertainly reduces to

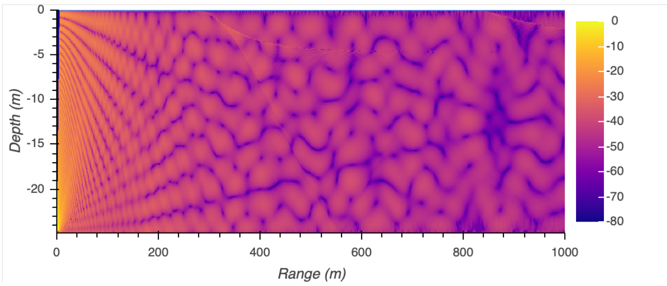

Fig. 1. Modeled transmission loss for the simulated environment

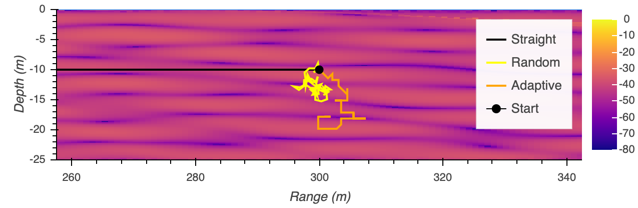

Fig. 2. A sample trajectory generated by each of the three policies

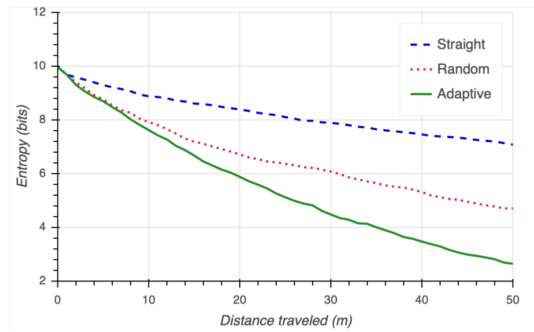

Fig. 3. Evolution of overall entropy of source location for the three policies

the measurement uncertainty (entropy of the measurement noise). Hence the information gained closely follows the prior uncertainty of modeled sound intensity, given $f(\boldsymbol{x}|\mathcal{Y}_i)$. We use weighted variance as a measure of prior uncertainty and choose an action at each time step to yield the next way point:

$$\begin{aligned} \boldsymbol{w}_{i+1} &= \arg\max_{\boldsymbol{w}\in\mathcal{A}(\boldsymbol{w}_i)} \sum_{\boldsymbol{x}} f(\boldsymbol{x}|\mathcal{Y}_i)(Z(\boldsymbol{w},\boldsymbol{x})-\mu)^2, \quad (2) \\ \mu &= \sum_{\boldsymbol{x}} f(\boldsymbol{x}|\mathcal{Y}_i) Z(\boldsymbol{w},\boldsymbol{x}), \end{aligned}$$

where $Z(\boldsymbol{w},\boldsymbol{x})$ is the modeled acoustic intensity at location $\boldsymbol{w}$ if the source is assumed to be at location $\boldsymbol{x}$, and $\mathcal{A}(\boldsymbol{w}_i)$ is the set of feasible moves for the robot.

The essential idea is to let the robot move in a direction where there is more uncertainty in modeled sound intensity values based on the probability distribution of source location, so as to maximize reduction of overall entropy. In summary, our algorithm iterates over two steps until the source location is confirmed to a required level of accuracy. The first step is to take a measurement at current robot's location and update the probability distribution of the source location using (1). The second step is to determine the optimal direction for the robot to move based on the weighted variance using (2).

III. Results

We illustrate the efficacy of our algorithm on a specific marine application of localizing a 1 kHz acoustic source on the seabed using an autonomous underwater vehicle (AUV) equipped with a single hydrophone. We simulate a 2D environment with 25 m water depth and 1 km range of interest. We assume a flat bathymetry, sandy seabed, and slightly downward-refracting sound speed profile. Fig. 1 shows the resultant transmission loss pattern for such an environment using a Bellhop underwater propagation model. Measured intensity values are simulated by adding random noise with a standard deviation of 10 dB to the modeled value in dB.

We start the AUV at a range of 300 m from the source and a depth of 10 m. Three different path planning policies are compared. The policy "straight" moves the AUV in a straight line at constant depth. The policy "random" moves the AUV in a randomly generated direction after each step (step size of 1 m). The last policy "adaptive" is our proposed algorithm to move the AUV in the direction that minimizes source localization entropy. The simulation for each policy allows the AUV to execute a 50 m path (50 time steps), and the simulation is repeated 100 times to collect performance statistics.

Fig. 2 shows a sample trajectory of the AUV generated by each of the three policies. The source localization entropy, averaged over 100 runs for these three policies, is plotted in Fig. 3 as a function of distance traveled. Table I summarizes the 90%-trimmed root-mean-square (RMS) localization error over the 100 runs. As clearly seen, our proposed adaptive policy results in the lowest overall entropy and smallest localization error among the three tested policies.

TABLE I
RMS localization error for the three policies

Policy	RMS Error
Straight	209.3 m
Random	52.2 m
Adaptive	10.6 m

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